Math 294 Week 6

2/19/2019 or 2/21/2019

This worksheet will emphasize relations. Informally, a *relation* is any symbol or property that conveys some relationship between groups of objects.

Here are some examples:

1. $\leq$ is a relation on $\mathbb{R}$ because $x \leq y$ conveys some relationship between the real numbers x and y; namely, that x is to the left of y on the number line.

2. $\subseteq$ is a relation on sets because $X \subseteq Y$ is telling us that all of X's elements are in fact elements of Y.

3. $\in$ is a relation because $x \in A$ is saying that x is an element of A.

To movivate the definition of a relation, consider the set $A = \{1, 2, 3, 4, 5\}$. Elements of A can be compared to each other by the symbol "$<$." Imagine you had to explain this ordering to a friend who had never seen the numberline before. You might consider writing down for your friend the following set: $R = \{(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}$.

The set R encodes the meaning of the $<$ relation for elements in A. In other words, an ordered pair (a, b) appears in the set if and only if $a < b$.

This motivates the following definition:

Definition. A relation R on a set A is *any* subset of ordered pairs of elements of A. In symbols, $R \subseteq A \times A$.

Problem 1. Let $A = \{0, 1, 2, 3, 4, 5\}$. Write out the relation R that expresses $>$ on A. Then illustrate it with a diagram.

Problem 2. Let $A = \{1, 2, 3, 4, 5, 6\}$. Write out the relation R that expresses $|$ (divides) on A. Then illustrate it with a diagram.

Problem 3. Let $A = \{0, 1, 2, 3, 4, 5\}$. Write out the relation R that expresses $\geq$ on A. Then illustrate it with a diagram.

Problem 4. Given a finite set A, how many different relations are there on A?

Problem 5. Consider the subset $R = (\mathbb{R} \times \mathbb{R}) - \{(x, x) \colon x \in \mathbb{R}\}$. What familiar relation on $\mathbb{R}$ is this? Explain.

Challenge Problem (not for submission): White to Play and Mate in 2. Write down all important variations.