

You may use results from the textbook, the online notes, and notes you took in class. You may not simply refer to or copy proofs from these sources.

1. Let G be a finite group and X a finite set with at least two elements. Suppose $G \times X \rightarrow X$ is a transitive group action. Show that there exists a $g \in G$ such that $X^g = \emptyset$, that is, for all $x \in X$ we have $g \cdot x \neq x$. (Hint: compare problem 2.) (20pts)
2. Let G be a finite group and H a proper subgroup of G (that is, $H < G$ but $H \neq G$). Show that G is not the union of the conjugates of H . In other words, prove that there is an element $x \in G$ such that for all $g \in G$ we have $x \notin gHg^{-1}$. (Hint: compare problem 1.) (20pts)
3. Let $p < q$ be primes. Suppose G is a finite group of order pq^n for some $n \geq 0$. Show that G is solvable. (10pts)