Math 547 **Homework 6** **Due November 9, 2007**

1. This exercise is about spheres. (15pts)

(a) Give a CW structure on the spheres S^n ($n \geq 0$) that has exactly two k-cells for every $k \leq n$ and such that the k-skeleton is homeomorphic to S^k. Work inductively (that is, explain how to attach two n-cells to S^{n-1} with its previously defined structure to obtain S^n.) Draw a picture for $n \leq 2$.

(b) Write down the cellular homology complex for S^2 with the CW-structure defined in (a) explicitely; that is, write down the groups $C_k(S^2)$ and the differentials. Use this complex to compute $H_*(S^2)$ (of course, we've done this before and actually use this previous computation in the construction of the cellular homology, but do the calculation anyway.)

(c) Let $S^\infty = \bigcup_{n\geq 0} S^n$ be the "infinite sphere", that is, the CW complex whose n-skeleton is S^n with the structure defined in (a). Compute $H_*(S^\infty)$.

2. This exercise is about degrees. Recall that a map $f : S^n \to S^n$ has degree $\deg(f)$ if $f_*([S^n]) = \deg(f)[S^n]$ where $[S^n]$ is a fixed orientation or fundamental class of the n-sphere. (20 pts)

a) Show that the map $s_0 : S^n \to S^n$ defined by $s(-x_0, \ldots, x_n) = (x_0, \ldots, x_n)$ has degree -1. Conclude tat the same is true for the map s_i that changes the sign of the coordinate x_i.

b) Show that for $f : S^n \to S^n$ and $g : S^n \to S^n$, we have $\deg(gf) = \deg(g)\deg(f)$.

c) Show that the degree of the antipodal map of S^n is $(-1)^{n+1}$.

3. Show that there is no retraction of the n-cell onto the $(n-1)$–sphere. That is, show that there cannot be a continuous map $p : D^n \to S^{n-1}$ such that $p|S^{n-1} = id$. (10pts)

4. Show that any continuous map $f : D^n \to D^n$ has a fixed point, that is, there is a point x such that $f(x) = x$. (15pts)