Mathematical Empiricism and its Role in Education: a Case Study

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The Focus on Mathematics Academy

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is work in progress

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is work in progress
over the past 25 years . . .

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and still going strong!

THE FOCUS ON MATHEMATICS ACADEMY

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Core People Involved

Al Cuoco (Education Development Center)
Wayne Harvey (Education Development Center)
Steve Rosenberg (Boston University and MfA Boston)
Glenn Stevens (Boston University and MfA Boston)

and many others . . .

The Focus on Mathematics Academy

The Focus on Mathematics Academy is a Collaboration of

- Boston University, Boston College, UMass Lowell
- Education Development Center, Inc. (EDC)
- Math for America Boston (MfAB)
- Boston area school districts

working to support a community of

- Teachers,
- Educators, and
- Mathematicians

to close the gap between school mathematics and mathematics as a scientific discipline.

The Focus on Mathematics Academy

A community of mathematical practice that has been evolving for almost 25 years.

- A Targeted Math and Science Partnership (MSP)
  - established in 2003
  - with funding by the National Science Foundation
- Rooted in *PROMYS for Teachers*, initiated in 1989
- Phase II research beginning in 2009
- DRK12 funding to develop measures of secondary teachers' algebraic habits of mind
- Noyce Projects at Boston University and Boston College with MfA Boston in support of Master Teacher Leaders

WORK TO DATE: *Focus on Mathematics* PROGRAMS

The Focus on Mathematics Academy has established

- **Study Groups**
- **Seminars and colloquia**
- A graduate degree program at Boston University
  - Master of Mathematics for Teacing (MMT)
- Summer Institutes – e.g. **PROMYS**
- Mathematics fairs and Mathematics Expo for students
- Research collaboratives
- Tools for Assessing Secondary Teachers Algebraic Habits of Mind
- **Teachers in leadership roles throughout**

How do we define effective teaching?

"An effective mathematics curriculum is one that

- pays attention to students,
- is rich with mathematics, and
- finds a way of connecting the two."

–William J. McCallum, in testimony to the National Academy of Sciences

But how we implement this depends on our beliefs

- about students and how students learn,
- about mathematics, and
- about connections between the two.

Beliefs about students & student learning

- All students can achieve at high levels in mathematics;
- Students can enjoy doing mathematics;
- Effective teaching requires:
  - insight into how students think/reason/learn and solve problems;
  - understanding the “meaning” of student questions and developing strategies for mining student ideas.

Beliefs about the nature of mathematics

- Mathematics is natural
  - The empirical nature of mathematics
  - People do mathematics naturally
- Mathematics exists independent of us
  - We can perform experiments
  - We can test ideas and decide for ourselves
- Experience precedes formality
  - “Meaning” is determined by experience
  - Definitions and theorems are capstones
  - Language is a tool for coming to terms with experience
- Mathematics is the science of structure
  - Operations, order
  - Shape
  - Continuity
  - Transformation
- Mathematics is the art of figuring things out

The Common Core State Standards in Mathematics

Examining the whole mathematical enterprise

- as a coherent body of knowledge
- as a way of thinking and inquiring about the world we live in

Corresponding to these, there are two parts to the CCSSM:

- Content Standards
- Practice Standards

AUTHENTIC MATHEMATICAL EXPERIENCES

Experience first:

"It has been observed in every human activity experience comes first, and as this experience grows the need for communication motivates the development of language. Sadly enough, in our classroom practice we place language first and experience second. We worry about what we should say in order to help the student 'understand.' By this we mean to provide the effect of experience through the use of suitably chosen words. Not unexpectedly, the effect is at best a very pale image of the real thing."

Arnold Ross

Typical features of an FoM mathematical experience

- Engagement in mathematics
- Teachers working together with each other and with mathematicians
- The central role of experience
  - empirical basis of mathematical knowledge
  - personal experience as guide for exploration
- Sharing ideas with others
  - in writing
  - in seminars
- Questioning answers
- Low threshold – high ceiling

On the correlation of mathematics with science

John Dewey's Laboratory School at the University of Chicago introduced an experiential philosophy of education based on "hands-on" experiences at the *elementary* level.

But, in 1899, Dewey wrote:

"The first person who succeeds in working out the real correlation of mathematics with science and advanced form of manual training, will have done more to simplify the problems of *secondary* education than any other one thing that I can think of."

Induction versus Deduction

(1) E.H. Moore's Laboratory Method

"Why should not the student be directed each for himself to set forth a body of geometric fundamental principles on which the would proceed to erect his geometric edifice? This method would be thoroughly practical and at the same time thoroughly scientific. The various students would have different systems of axioms, and the discussions thus arising in the minds of all: precisely what are the functions of the axioms in the theory of geometry."

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(2) **Charles Dodgson's theorem:**
Every triangle is isosceles!

Program in Mathematics for Young Scientists (PROMYS)

PROMYS for Students

- Founded in 1989
- Inspired by the Ross Program (going back to 1957)
- Underlying principles are to be found in the earlier work of E. H. Moore.

PROMYS for Teachers

- Founded in 1991
- The flagship program of the Focus on Mathematics Academy
- Teachers work alongside students on significant mathematics

GUIDING PRINCIPLES OF PROMYS

PROMYS offers an **immersion experience** of mathematics (not a teaching experience).

- **Everyone is a mathematician**
- Intensely personal engagement in mathematics
- Depth over breadth
- Experience before formality
- Necessity for communication
- Innovation and independence of thought, not just in mathematics
- **Growth within a multi-layered community of "mathematician/teachers"**

THE PROMYS COMMUNITY (2013)

- First year participants
  - 60 high school students
  - 23 teachers
- Returning participants
  - 23 high school students
  - 12 teachers
- Counselors
  - 22 undergraduates
  - 9 teacher alumni and 4 graduate students
- Faculty
  - 7 mathematicians (for seminars)
  - 6 mathematicians (as outside research mentors)

THE PROMYS PROBLEM SETS

Design of the Problem Sets

- Numerical problems
  - the most important problems on each set
  - accessible to everyone
  - a mathematical laboratory
  - exercise powers of perception
  - use language as a tool to understand experience
- Prove or disprove and salvage if possible
  - using language as a tool for exploration
  - a dialectic between experience and formalism
- Other problems
  - Exploration
  - Asking questions
  - Technique of generalization
- A rich network of problem threads develop across problem sets to create opportunities for condensation of ideas.

Supporting Elements

The Counselor Staff

- Counselors are the heart and soul of PROMYS
  - Engaged in developing their own mathematical careers as researchers and/or educators
  - Role models engaged in their own learning . . . not teachers!
  - Respond to questions with more questions to suggest productive lines of exploration for the participants.

The Morning Lecture

- Support for participant understanding of the experiences they find in the problem sets
  - Trail participant work on the problem sets
  - Summarize and tie together threads in the problem sets
  - Provide broader context for understanding

Axioms, Definitions, and Theorems as a Reduced Inventory

Three "phases" of investigation

(1) (Naiveté) A vast and chaotic supply of experience
- Naive experience leads to intuition.
- Powers of observation are refined.
- Formulate conjectures and design experiments to test them.

(2) (Formalism) Language as an organizational tool.
- Abstraction as condensation of ideas.
- Definitions arise naturally as we articulate what we see/feel.
- Axioms and theorems build a reduced inventory
- Proofs as threads connecting different parts of experience

(3) (Critique) Seeking generative power from our ideas
- Refinement of ideas
- Generalization as a tool for developing new experiences

THANK YOU!

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Visit our websites:
www.focusonmath.org
www.promys.org

Visit us in Boston this summer:
June 29 to August 9, 2014