Arctic Sea Ice and Linear Equations

Background

European explorers, beginning with Cabot's 1497 attempt to sail to the Orient from England, searched for the *Northwest Passage*, a route through the Arctic Ocean along the coast of Canada. See Figure 1. The Norwegian explorer Roald Amundsen was the first to complete the journey, though it took from 1903 to 1906. In 1957, the U.S. Coast Guard Cutter *Storis* became the first U.S. vessel to circumnavigate the North American continent, a 22,000 mile trek.

The problem is the Arctic Ocean is covered by a sea ice pack nearly all the time---the passage is closed. Since the beginning of the Industrial Revolution, global temperature averages have risen overall causing more of the ice pack to melt in the summer. NASA's *National Snow and Ice Data Center* at the University of Colorado, Boulder, has collected data provided by satellites, over-flights, submarines, and other observations measuring the amount of sea ice in the Arctic Ocean for several decades. The amount of ice is at a minimum in September, the end of summer. Figure 2 shows the September average extent of total Arctic sea ice area in millions of square kilometers versus the year from 1979 to 2011. Clearly the extent of ice is decreasing.

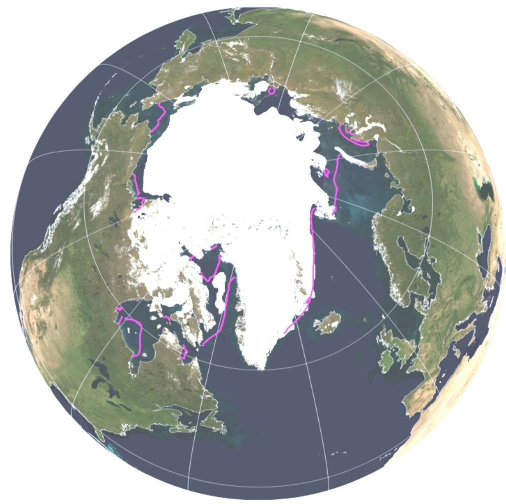

Figure 1: Arctic Sea Ice[1]

Melting sea ice provides an example of a positive feedback loop (also called non-linear feedback). As the ice melts, it leaves more ocean open. Ice is very reflective giving the arctic region a high *albedo*; ice reflects up to 70% of the sun's energy. The ocean is darker, reflecting only 6% of the sun's energy, so as the ice pack retreats, the area's albedo gets lower. More energy is absorbed by ocean water than by sea ice increasing the temperature, causing more ice to melt leading to more open water, creating a positive feedback loop.

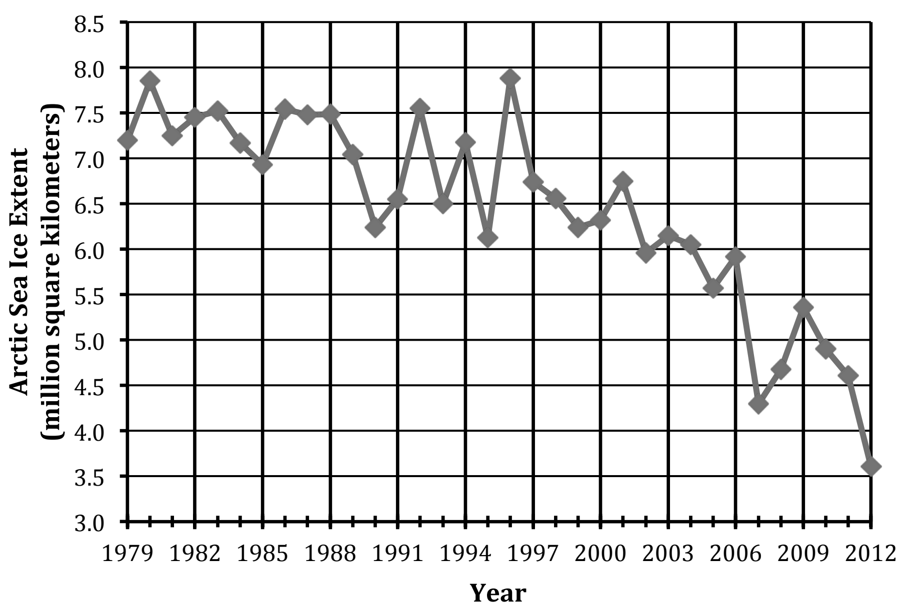

Figure 2: Average September Arctic Sea Ice Extent, 1979 to 2012.

Questions:

1. From the data given in Figure 2, how much ice was in the Arctic in September 1988, September 1998 and September 2008? What are the units? What is the overall trend in the amount of sea ice during this time period?

2. On Figure 2, draw the line that looks to give the best fit to the data.

3. From your drawing, calculate the slope of the line. How does your estimate connect with the overall trend?

4. Figure 3 shows both the equation and the graph of the line of best fit: $y = -0.0921\ t + 190.12$ Use this mathematical model (the equation of the line of best fit) to find the amount of ice in September 2003 and in September 2012. How do these values compare to the actual data values?

5. What is the slope of this line? How close was the slope of your line in (2) to the slope of the best fit line?

6. What are the units for the slope? What does the slope tell us about the rate at which extent of sea ice is changing?

7. If present trends continue, how much Arctic sea ice will there be in September 2020? In what year does your model predict that the Arctic will be ice free in September?

8. Looking at the data, do you think this prediction is accurate? Discuss.

9. What do you consider as the pros and cons of having an open Arctic passage?

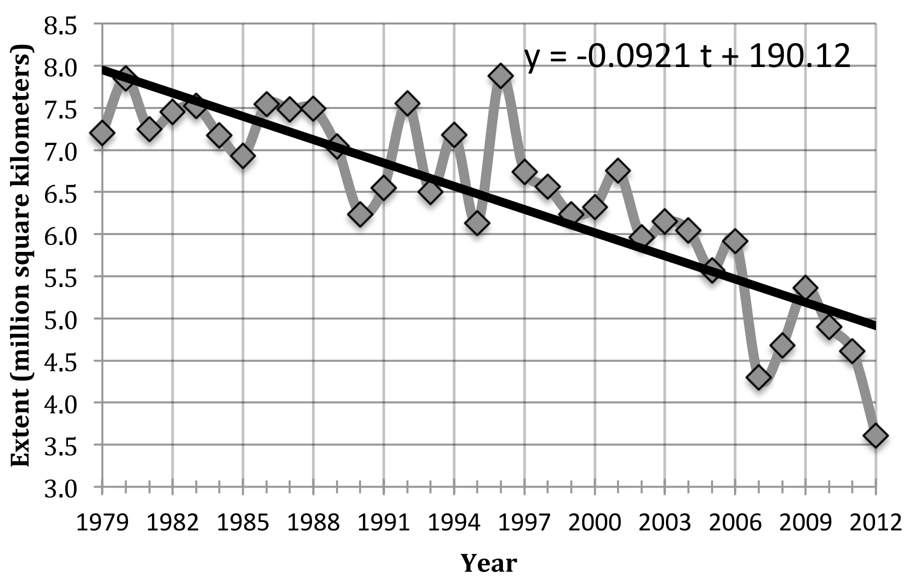

Figure 3: Best fit curve to Average September Arctic Sea Ice Extent

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[1] All images courtesy of NASA and the National Snow and Ice Data Center, U.C., Boulder. (http://nsidc.org). Lesson plan by William Bauldry, Appalachian State University, Victor Donnay, Bryn Mawr College, and Thomas J. Pfaff, Ithaca College.

Arctic Sea Ice and Linear Equations
Teacher Guide

William C. Bauldry, Appalachian State University,
Victor J. Donnay, Bryn Mawr College, Thomas J. Pfaff, Ithaca College

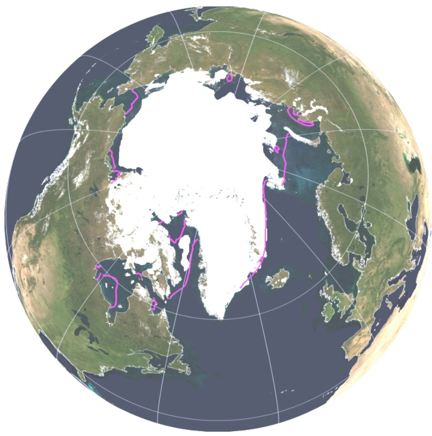

Figure 1: Arctic Sea Ice

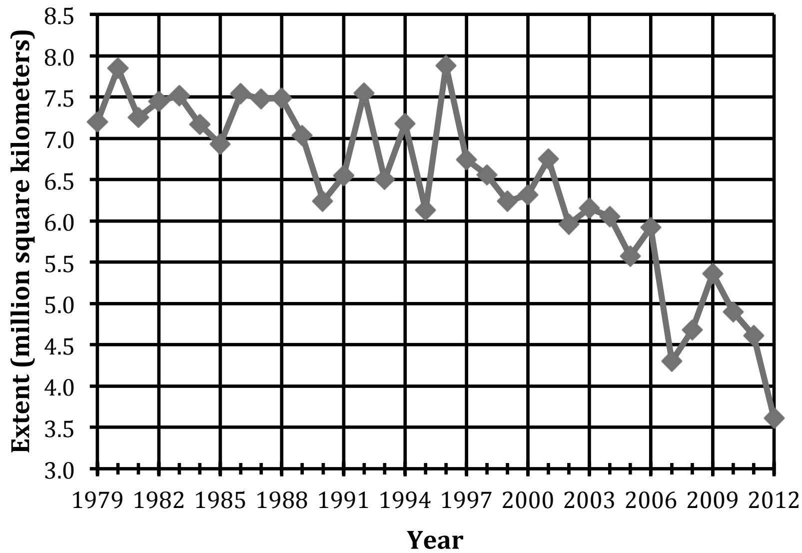

Figure 2: Average September Arctic Sea Ice Extent, 1979 to 2012

Lesson Overview: We examine the extent of sea ice in the Arctic (see Figure 1) using images from NASA and data from the National Snow and Ice Data Center, U.C., Boulder [1] (see Figure 2). Students will use a line of best fit to model this data and then use the mathematical model to predict future sea ice levels. Specifically they will predict when the Arctic will start becoming ice free.

Level: Algebra 1 and above.

Common Core State Standards for Mathematics [2]: Standard for Mathematical Practice 4 - Model with mathematics and High School: Algebra >> Creating Equations standards.

Associated Materials: The spreadsheet Sea Ice Data contains the data used to create the graph as well as the linear fit to the data.

Mathematical Content:

1. Graphs. Graphs provide information about the real world. In this case they show the average extent of the Arctic sea ice in September (measured in millions of square kilometers) as a function of time (measured in years). Graphs can be generated by data that is given in a table.
2. Line of best fit. Given a set of data points, one can find a line that fits the data as well as mathematically possible.
3. Mathematical modeling. The line of best fit is described by a formula or function. This function provides what mathematicians call a mathematical model of the extent of sea ice. One can use a mathematical model to predict the future – in this case the future extent of the Arctic sea ice.
4. Properties of graphs. Graphs can be increasing or decreasing and be concave up or down. These properties have important contextual implications.
5. Slope and rate of change. The slope of a line can be interpreted as the rate of change of the function.
6. Units. Units are important in the mathematics of real world problems and keeping track of units can help one better understand mathematical concepts. The units of rate of change in this problem are million km^2 per year = million km^2/year which has a crucial meaning in the context of the problem. It tells how much less ice there will be in each successive year.

Sustainability Content:

European explorers, beginning with Cabot's 1497 attempt to sail to the Orient from England, searched for the *Northwest Passage*, a route through the Arctic Ocean along the coast of Canada. See Figure 1. The Norwegian explorer Roald Amundsen was the first to complete the journey, though it took from 1903 to 1906. In 1957, the U.S. Coast Guard Cutter *Storis* became the first U.S. vessel to circumnavigate the North American continent, a 22,000 mile trek.

The problem is the Arctic Ocean is covered by a sea ice pack nearly all the time---the passage is closed. Since the beginning of the Industrial Revolution, global temperature averages have risen overall causing more of the ice pack to melt in the summer. NASA's *National Snow and Ice Data Center* at the University of Colorado, Boulder, has collected data provided by satellites, over-flights, submarines, and other observations measuring the amount of sea ice in the Arctic Ocean for several decades. The amount of ice is at a minimum in September, the end of summer. Figure 2 shows the September average extent of total Arctic sea ice area in millions of square kilometers versus the year from 1979 to 2011. Clearly the extent of ice is decreasing.

Melting sea ice provides an example of a positive feedback loop (also called non-linear feedback). As the ice melts, it leaves more ocean open. Ice is very reflective giving the arctic region a high *albedo*; ice reflects up to 70% of the sun's energy. The ocean is darker, reflecting only 6% of the sun's energy, so as the ice pack retreats, the area's albedo gets lower. More energy is absorbed by ocean water than by sea ice increasing the temperature, causing more ice to melt leading to more open water, creating a positive feedback loop.

Lesson Plan Notes

This lesson can be taught in a variety of ways ranging from a teacher centered presentation to having students working through the assignment on their own or in groups and then presenting their findings to the class.

0. Lesson launch. Engage students in a discussion of Arctic sea ice. What do students know about this? What questions do they have? Students might know about the soda company initiative to save polar bears . Show some pictures or a short video about the issue (for example, the video *A New Climate State: Arctic Sea Ice 2012* [3]) . One might show some of the video at the start of class as an introduction and more at the end as a wrap up.

1. The Graph. Understanding and paying attention to the units is important. Often in math problems, there are no units. So students do not always pay attention when there are units. The horizontal axis measures time in years. When answering the questions in the lesson, the students should always include the units. The vertical axis gives the average extent of Arctic sea ice in September. The units are square kilometers measured in millions (million km^2).

   An extension question, would be:

   Convert your answer from units of km^2 into units of $miles^2$ using the conversion factor 1 mile = 1.6 km.

   Students are likely to forget that that they will need to square the conversion factor since they are dealing with area:

$$\# \text{ km}^2 \times \left(\frac{1 \text{ mile}}{1.6 \text{ km}}\right)^2$$

2. Finding the line of best fit. When doing this by hand, there will be a lot of variation in the answers. As long as the students come up with something reasonable, that is fine. One can have an animated discussion with students explaining why they drew the line they did and seeing the variation among the results. This would provide a teachable moment for the need to have a precise definition of line of best fit.

3. Slope of line. The students will need to calculate the slope from the drawing by calculating $\text{m} = \frac{\Delta y}{\Delta t}$. The units for slope are given by the units for the ratio $\text{m} = \frac{\Delta y}{\Delta t} = \frac{\text{millions km}^2}{\text{year}}$. Many students are not used to dealing with slope in the context of units and could find this question challenging.

4. In Excel, we used the Chart – Add Trendline feature and found that the line of best fit is given by y = -0.0921 t + 190.12.

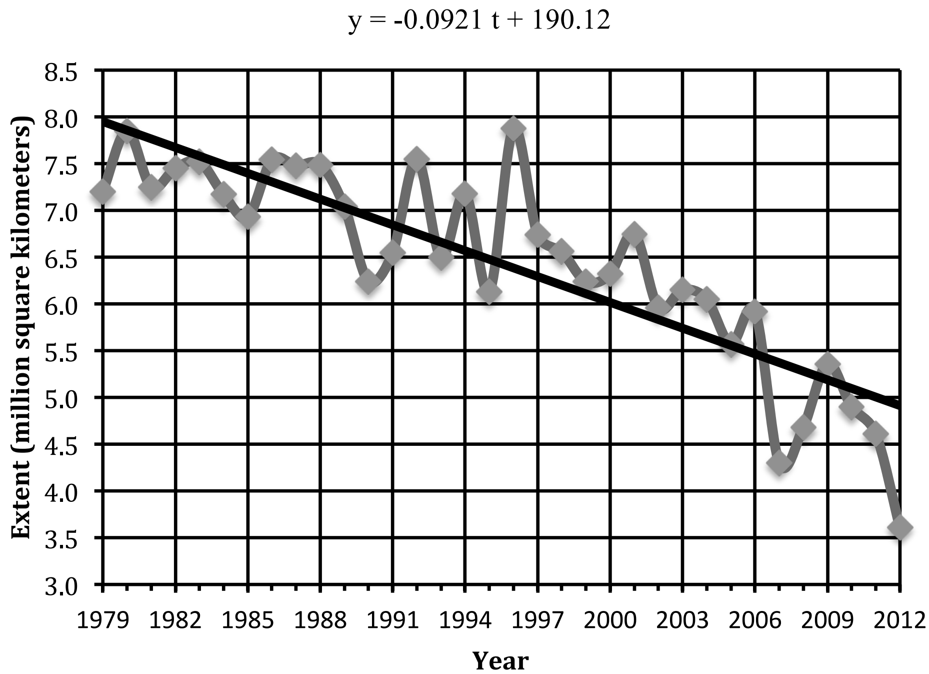

Figure 3: Best fit curve to Average September Arctic Sea Ice Extent

In 2003, the line gives y = -0.0921 (2003) + 190.12 = 5.64 million km^2 and in 2012, y = -0.0921 (2012) + 190.12 = 4.81 million km^2.

5 - 6. The slope is m = -0.0921, approximately -0.09, with units of million km^2 per year. This means that each year, the amount of sea ice in September will be 0.09 million km^2 less than the year before.

7. Use mathematics to predict the future: Using the model students need to find y when t = 2020:

$$y = -0.0921\,(2020) + 190.12 = 4.08 \text{ million } km^2.$$

   To determine when the Arctic will be ice free, the students must solve the equation

$$y = -0.0921\,t + 190.12 = 0$$

   which gives t = 2064.

8. The data in the last three years are all below the line of best fit, with the most recent values of 2012 being significantly below the line. So the best fit line could be a poor predictor of the future because it weights earlier years equally with recent years. But the recent years suggest that the behavior of the sea ice is changing. The positive feedback loop might be accelerating the melting. The problem of sea ice melt is very complex due to the non-linear feedbacks involved; the scientific community has not reached consensus on when they think the ice free stage will arrive [4].

9. Some pros of having an ice free passage through the Arctic sea: it will make shipping through the Northwest Passage possible, which will reduce cost and travel time for

goods. It might also give access to more natural resources. Cons include that will make it difficult for polar bears to survive. And it is an indicator that serious climate changes are taking place on the earth and these changes will have far reaching consequences.

Extension Questions:

1. Using the data for just the past several years (ex. 2009-2012), predict what the extent of Arctic sea ice will be in 2013 and 2014 and predict when the Arctic will be ice free in September. Using just the more recent data will lead to a significantly shorter estimate.

2. In Figure 4 (see next page) we give the extent of Arctic sea ice in March when the ice extent is near its maximum. In Figure 5, we give the linear best fit. Note that here we the units for time are years after 1970. For this second set of data, have the students repeat questions 1 – 8. The students can then discuss the differences in their predictions for the two different times of year.

References:

1. National Snow and Ice Data Center, U.C., Boulder, at http://nsidc.org.

2. Common Core Standards at http://www.corestandards.org/Math ; http://www.corestandards.org/Math/Practice/MP4 ; http://www.corestandards.org/Math/Content/HSA/CED

3. *A New Climate State: Arctic Sea Ice 2012,* at http://www.youtube.com/watch?v=ZYaubXBfVqo&feature=player_embedded

4. When will the Arctic be ice-free in the summer? Maybe four years. Or 40. Brad Plume at Wonkblog, http://www.washingtonpost.com/blogs/wonkblog/wp/2012/09/20/when-will-the-arctic-be-ice-free-maybe-four-years-or-40/

Acknowledgements:

The lesson was developed in conjunction with the MAA PREP workshop: Educating with Math for a Sustainable Future workshop. Lynn Foshee Reed provided valuable feedback on the lesson.

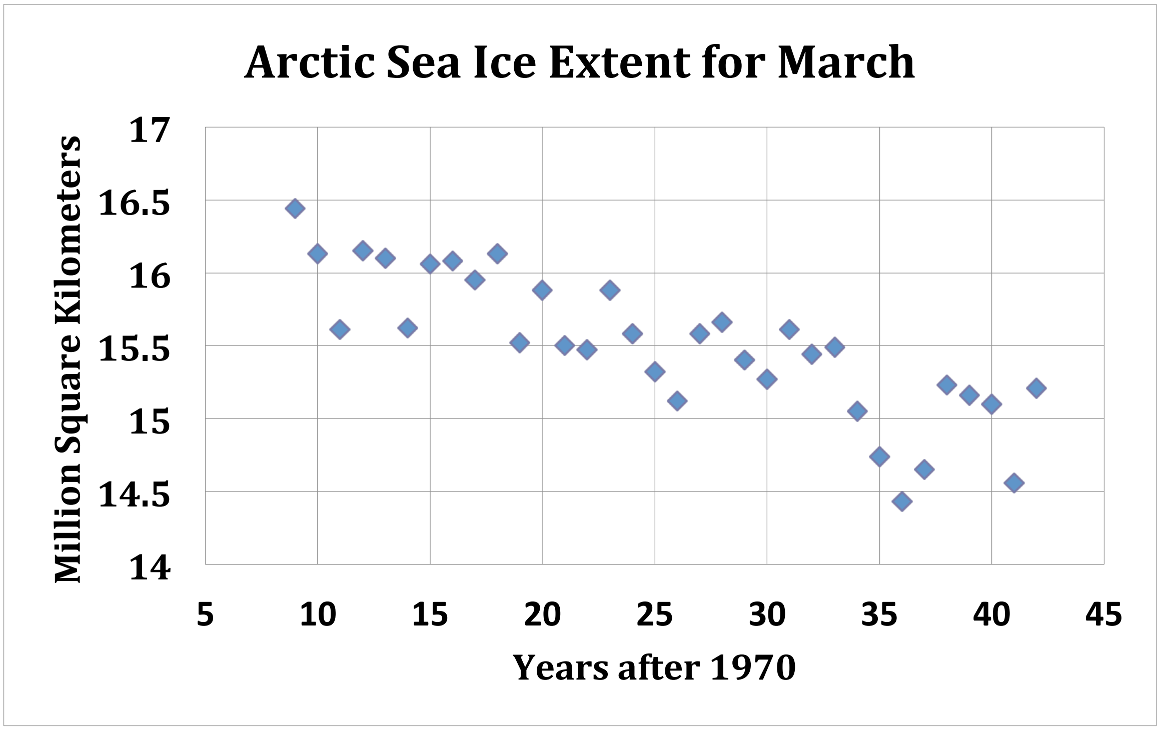

Figure 4: Average March Arctic Sea Ice Extent, 1979 to 2012.

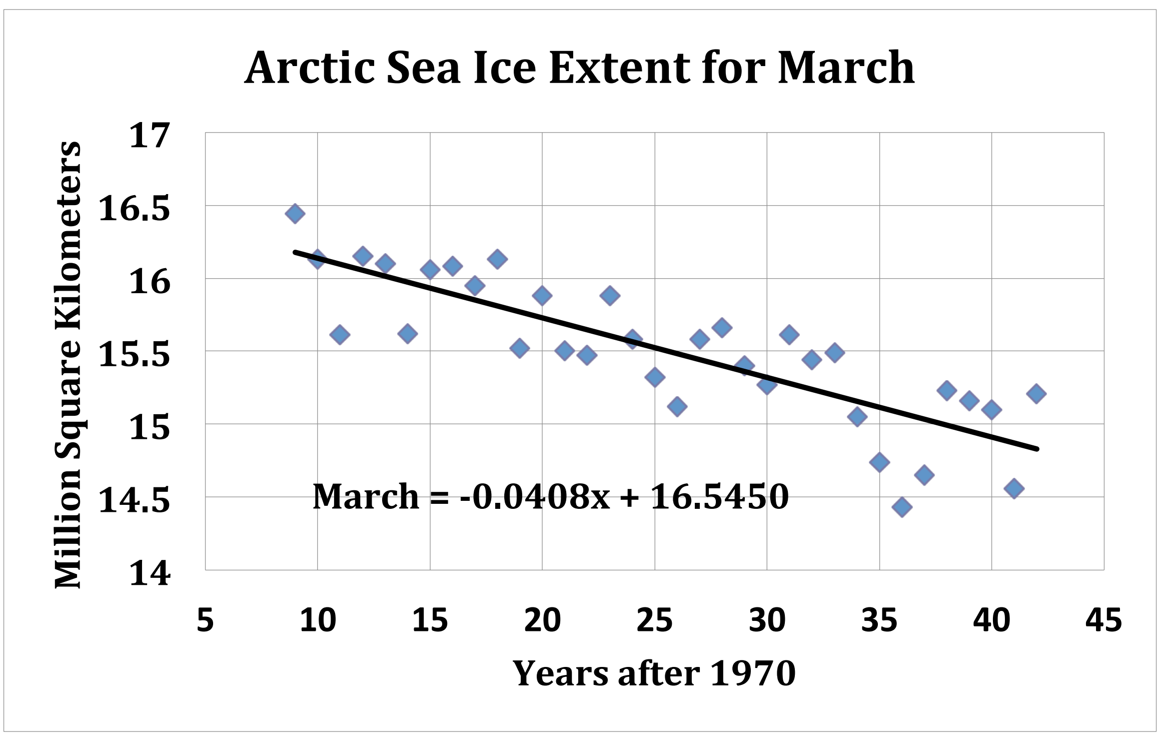

Figure 5: Best fit curve to Average March Arctic Sea Ice Extent.

Background

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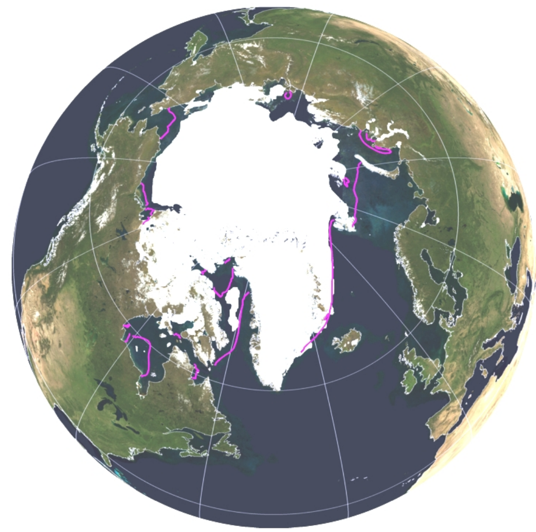


Figure 1: Arctic Sea Ice[1]

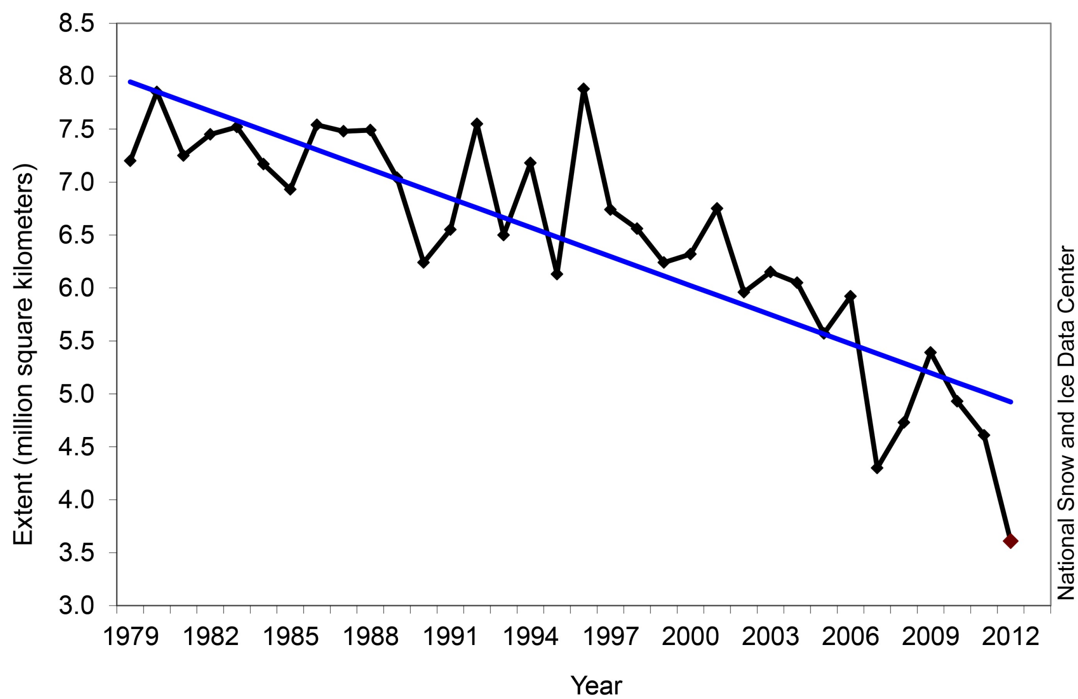


Figure 2: Average September Arctic Sea Ice Extent, 1979 to 2012

[1] All images courtesy of NASA and the National Snow and Ice Data Center, U. C., Boulder. (`http://nsidc.org`).

Lesson plan by William C. Bauldry, Appalachian State University.

Project

Determine the overall trend in the average monthly sea ice extent using the data given in Table 1.

1. List the month __________ your group is assigned.
2. Use NSIDC's *Arctic Sea Ice Extent Averages* data given in Table 1 for your assigned month to derive a linear function giving the overall trend of the average sea ice extent.
3. Plot the linear function and your data on the same graph.
4. Predict your month's 2013 and 2014 values.
5. Explain why the slope of the linear function describes the trend of your data. What is the trend as a percentage?
6. Is the trend you found reasonable? Why or why not?

Extra for Experts: Explore quadratic models of the data.

Table 1: **MONTHLY ARCTIC SEA ICE EXTENT AVERAGES** (million square km.)

Year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
May	14.06	14.04	13.9	14.17	13.54	13.68	14.23	13.52	13.81	13.69	12.98	13.3
June	12.59	12.31	12.57	12.69	12.35	12.20	12.40	12.10	12.57	12.02	12.31	11.68
July	10.47	10.39	10.62	10.75	10.91	10.15	10.09	10.47	9.98	10.04	10.38	9.62
Aug	8.15	8.04	7.86	8.26	8.36	7.87	7.46	8.01	7.69	7.90	7.92	6.82
Sept	7.20	7.85	7.25	7.45	7.52	7.17	6.93	7.54	7.48	7.49	7.04	6.24
Oct	9.39	9.46	9.19	9.98	9.64	8.84	8.88	9.89	9.29	9.47	9.52	9.35

Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
May	13.51	13.25	13.54	13.73	13.04	13.06	13.32	13.80	13.86	13.18	13.72	13.12
June	12.23	12.13	11.99	12.10	11.55	12.10	11.91	11.85	12.10	11.71	11.69	11.69
July	9.68	10.61	9.66	10.22	9.15	10.36	9.59	9.62	9.59	9.75	9.22	9.49
Aug	7.40	7.86	7.29	7.61	6.68	8.17	7.30	7.49	7.38	7.21	7.47	6.53
Sept	6.55	7.55	6.50	7.18	6.13	7.88	6.74	6.56	6.24	6.32	6.75	5.96
Oct	9.16	9.6	9.18	9.48	8.94	9.39	8.76	8.85	9.1	8.92	8.59	8.81

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
May	13.00	12.58	12.99	12.62	12.89	13.16	13.39	13.10	12.79	13.13		
June	11.77	11.51	11.29	11.06	11.49	11.46	11.49	10.87	11.01	10.97		
July	9.46	9.60	8.93	8.67	8.13	9.06	8.82	8.39	7.92	7.94		
Aug	6.85	6.83	6.30	6.52	5.36	6.06	6.26	5.98	5.52	4.72		
Sept	6.15	6.05	5.57	5.92	4.30	4.68	5.36	4.90	4.61	3.61		
Oct	8.65	8.48	8.45	8.33	6.77	8.42	7.52	7.71	7.10	7.00		

Source: NASA's National Snow and Ice Data Center, Univ. of Colorado, Boulder; http://nsidc.org/arcticseaicenews/

Report Requirements

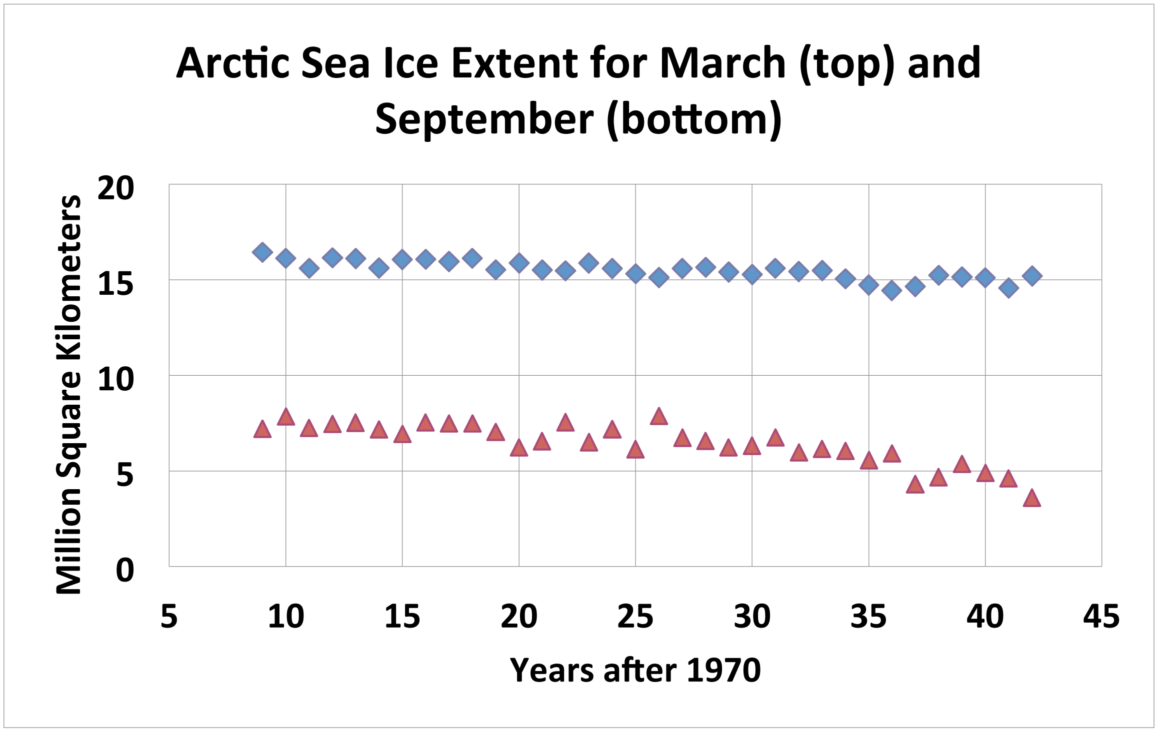
Your report must include:

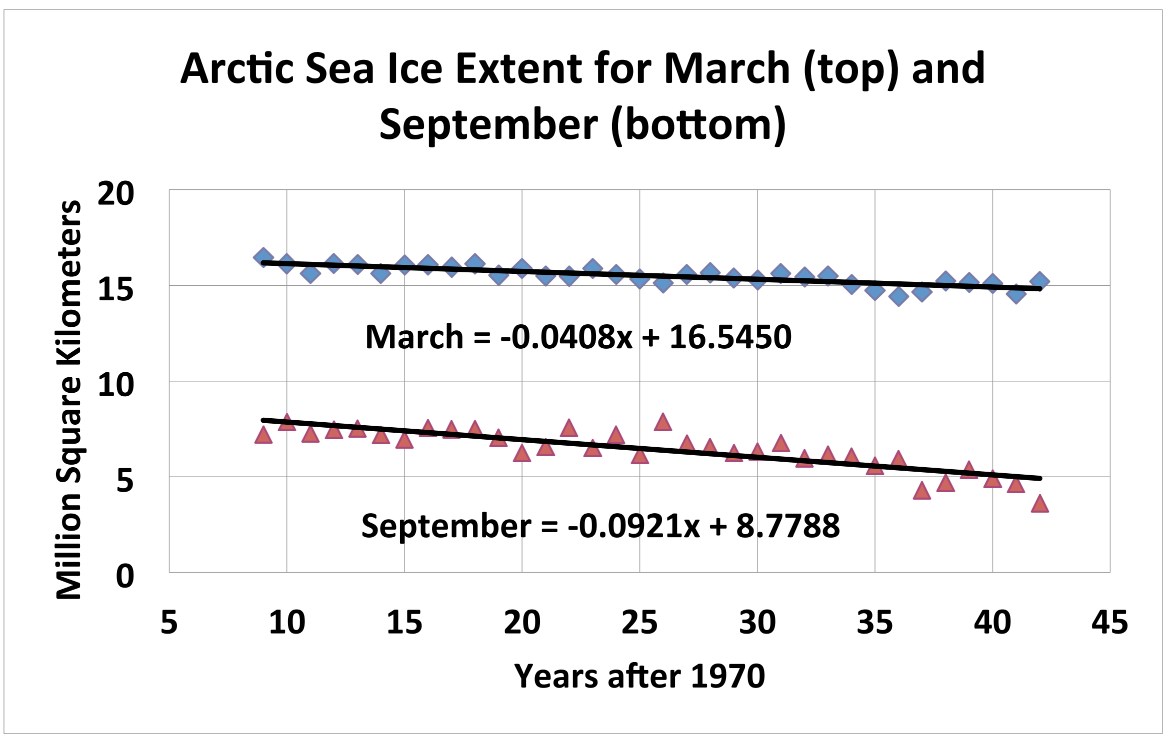
1. Your project team members' names.
2. Your linear model and predictions for 2013 and '14 with a justification of its appropriateness.
3. A graph showing your model with the data points.
4. A discussion of the trend you calculated and whether or not it reasonably describes the data.

Possible formats for your final report:

- a standard paper (in **pdf** format; **not** doc, docx, &c.)
- a slide-show (Impress, Keynote, Powerpoint, or pdf)
- a video of your team presenting to a group of classmates (in Quicktime or Windows Media format; mp4, mpeg, mov, wmv, or avi; **not** flv or swf)

Year	Years After 1970	March (in millions of sq km)	September (in millions of sq km)
1979	9	16.44	7.2
1980	10	16.13	7.85
1981	11	15.61	7.25
1982	12	16.15	7.45
1983	13	16.1	7.52
1984	14	15.62	7.17
1985	15	16.06	6.93
1986	16	16.08	7.54
1987	17	15.95	7.48
1988	18	16.13	7.49
1989	19	15.52	7.04
1990	20	15.88	6.24
1991	21	15.5	6.55
1992	22	15.47	7.55
1993	23	15.88	6.5
1994	24	15.58	7.18
1995	25	15.32	6.13
1996	26	15.12	7.88
1997	27	15.58	6.74
1998	28	15.66	6.56
1999	29	15.4	6.24
2000	30	15.27	6.32
2001	31	15.61	6.75
2002	32	15.44	5.96
2003	33	15.49	6.15
2004	34	15.05	6.05
2005	35	14.74	5.57
2006	36	14.43	5.92
2007	37	14.65	4.3
2008	38	15.23	4.68
2009	39	15.16	5.36
2010	40	15.1	4.9
2011	41	14.56	4.61
2012	42	15.21	3.61


Arctic Sea Ice Extent for March (top) and September (bottom)
Million Square Kilometers
20
15
10
5
0
5
10
15
20
25
30
35
40
45
Years after 1970


Arctic Sea Ice Extent for March (top) and September (bottom)
Million Square Kilometers
20
15
10
5
0
March = -0.0408x + 16.5450
September = -0.0921x + 8.7788
5
10
15
20
25
30
35
40
45
Years after 1970