Monte Carlo Simulation Project for Derivatives Markets Class

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Outline

- Group Discussion Part I
- The Monte Carlo Simulation Project (Independent study version)
- Group Discussion Part II

Group Discussion Part I

- What simulation experience did you have before (either as a student or as a teacher)?
- What was the most useful part about it?
- What difficulties did you encounter?

Monte Carlo Simulation Project

- Students use Monte Carlo simulation to estimate the European call option prices and Greeks.
- **Audience**: Actuarial science students. (Undergraduate honor student capstone project, senior student independent study, or Master students in Derivative Markets class)
- **Prerequisite**: Financial Mathematics, Probability
- **Contents**: Four Assignments + Poster Presentation
- **Period**: One Semester

Assignment One (Weeks 1-4)

Reading: *Derivative Markets* by Robert McDonald [1], chapter 12, chapter 18, and chapter 19 (19.1 – 19.5).

Topics: Black-Scholes formula, lognormal distribution, and Monte Carlo method.

Assignment: Students take detailed notes and respond to:

- What's the advantage of Black-Scholes model?
- What's the original formula for calculating the theoretical option price?
- What are the normal and lognormal distributions?
- Why is the lognormal distribution used to determine stock prices?
- What is Monte Carlo valuation?
- Why is Monte Carlo preferred for option pricing?
- What is the difference between Black-Scholes and Monte Carlo for option pricing?

Worth: 15% of the final project grade

Assignment Two (Weeks 5-6)

Topic: European call option pricing framework for both Black-Scholes and Monte Carlo methods.

Assignment:

1. Explain the definition of the European call option: the policyholder of the option has the right but not obligation to buy the underlying asset (e.g. a stock) at a fixed price (strike price) at maturity.

2. Give and explain the pricing formulas: The payoff function for the European call option is

$$Option\ Payoff = \max(0, S_T - K),$$

where S_t stands for the stock price at certain time t, T is the maturity time, and K is the strike price.

Worth: 5% of the final grade.

Quick introduction to options pricing

The lognormal stock price for simulation is given by

$$S_T = S_0 \exp\left(\frac{r-\sigma^2}{2} \times T + \sigma\sqrt{T}Z\right),$$

where S_T stands for the stock price at maturity, S_0 stands for the initial stock price; r stands for the risk-free interest rate, σ stands for the volatility of stock prices, T stands for maturity time, and Z is the generated standard normal random numbers.

Once the stock price at maturity is simulated, the averaged discounted payoffs will be the European call option price:

$$\bar{C} = \frac{\sum_{i=1}^{N} \max(0, S_T^i - K)}{N} e^{-rT},$$

where C stands for the option price, S_T^i stands for the i^{th} simulated stock price at maturity, N stands for the number of simulations for S_T, and other notations are the same as earlier defined.

Assignment Three (Weeks 7-8)

Topic: Basic stochastic differential equations

Assignment:

1. Derive (advanced) or understand (intermediate-level) the Monte Carlo pricing formula based on the lognormal model.

2. Determine parameters' values from actual financial resources (yahoo finance, research papers on options pricing, etc.)

3. Implement the Monte Carlo simulation in Excel with these parameters.

4. Give a table of results.

Worth: 25% of the final grade.

Useful Excel Commands

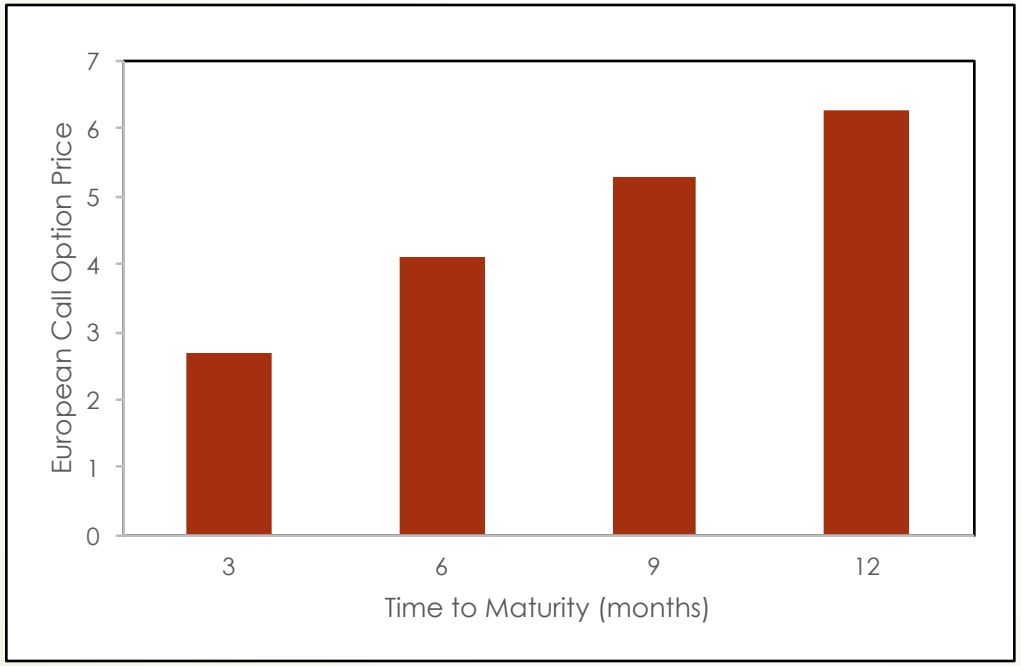
- **NORMSINV**(p) returns the value z such that, with probability p, a standard normal random variable takes on a value that is less than or equal to z
- **RAND**() Returns an evenly distributed random real number greater than or equal to 0 and less than 1. A new random real number is returned every time the worksheet is calculated.
- **EXP**(x) Returns $e^{x.}$
- **SQRT**(x) Returns the square root of x.
- **MAX**(a,b) Returns maximum of a and b.
- **AVERAGE**() Returns the **average** (arithmetic mean) of the arguments. For example, if the range A1:A20 contains numbers, the formula =**AVERAGE**(A1:A20) returns the **average** of those numbers.

(Definitions from Microsoft Support)

Sample Simulation Results

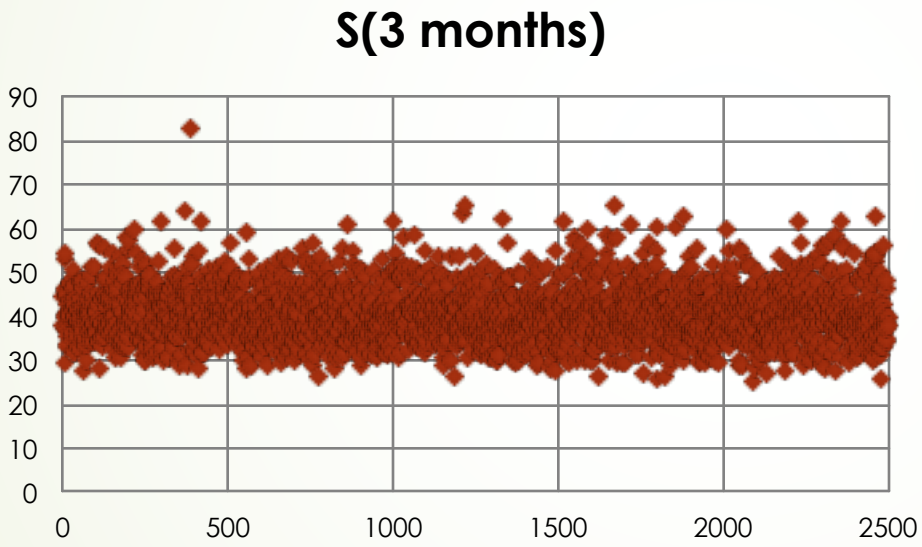
European call option prices for increasing time to maturity.

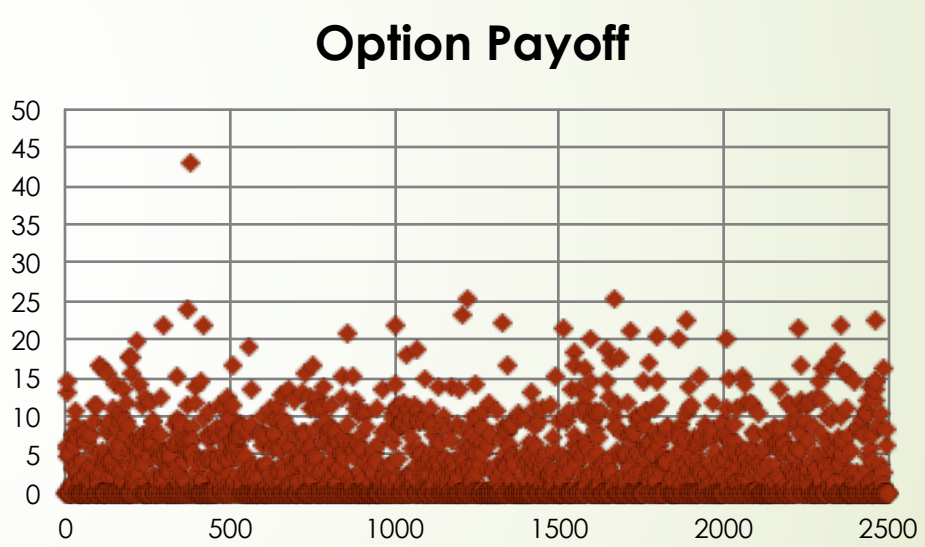
Time	Option Price
3 months	2.8188
6 months	4.2859
9 months	5.0138
12 months	6.4756



Activity: How does volatility influence the stock price and expected payoff?

Please open excel and create plots with different volatilities.





Assignment Four (Weeks 9-10)

Topic: Estimating Greeks: delta.

Reading: *Derivative Markets* Chapter 12 section 12.3 [1] (Greeks). *Monte Carlo Methods in Financial Engineering* by Paul Glasserman [2] sections 7.1 and 7.2 (Monte Carlo algorithm).

Background: Delta is defined by $\frac{dC}{dS_0}$, which measures the option price change when the stock price increases by $1. The pathwise Monte Carlo estimator for delta is:

$$Delta = e^{-rT}\frac{S_T}{S_0}1\{S_T > K\}.$$

Assignment: Simulate Delta in Excel with parameters from assignment 3, create table of results.

Worth: 15%

Simulation Results

Table 2 European call option delta values for increasing time to maturity.

Time	**Delta Values**
3 months	0.5815
6 months	0.6261
9 months	0.6430
12 months	0.6747

(Parameters used in simulation are from [1] example 19.1.)

Poster Presentation (Weeks 11-12)

The students are given a template. The poster includes:

- Introduction: the background of this project; definition of European call option.
- Monte Carlo Method: the definition and application of Monte Carlo simulation method; important formulas.
- Simulation results from assignments 3 and 4.
- Conclusion: advantages of Monte Carlo simulation; properties of European call option based on simulation results.

Worth: 40% of the final grade. Alternately, can be a paper.

Extensions

- 1. Extend the Monte Carlo Simulation to estimate European put option prices.
- 2. Estimate different types of Greeks which tell students how parameter changes affect the option prices in different ways.
- 3. Connect assignments to the MFE exam. The Society of Actuaries (SOA) Models for Financial Economics (MFE) exam questions relate to the project assignments. Each assignment can be assigned 2 – 3 problems in light of the matching content. For example, in the third assignment, we can put a problem which tests the Monte Carlo simulation algorithm or normal random numbers generating. The embedded problems will add value to MFE exam preparation.

Group Discussion Part II

- What Monte Carlo simulation or simulation projects you may create for your teaching?
- What resources or information would help you with this?

References

- 1. Derivatives Markets, Robert L. McDonald, 3rd edition.
- 2. Monte Carlo Methods in Financial Engineering, Paul Glasserman, 2004.

Thank You and Questions!