

## MATH 417: FINAL PROBLEM SET

Due Wednesday December 2 before class in Gradescope. You may use your class notes, previous homework and the course text book. You may not use any other materials, including other text books, the web, question centers, etc. The work should be yours and yours alone. Please do not collaborate.

**Problem 1.** (5 points) Find all the solutions of the equation  $z^5 = \frac{32}{\sqrt{2}} + \frac{32i}{\sqrt{2}}$ .

**Problem 2.** (5 points) Determine all the values of  $(ie^\pi)^{3i}$ . Indicate the principal value.

**Problem 3.** (10 points) Let  $u(x, y)$  and  $v(x, y)$  be a conjugate pair of smooth harmonic functions in a domain  $D$ . Prove that  $u^3 - 3uv^2$  is a harmonic function in  $D$ . Find all possible harmonic conjugates of  $u^3 - 3uv^2$ .

**Problem 4.** (10 points) Let  $f(z)$  be an entire function. Suppose that for  $z \neq 0$ ,

$$f(z) = f\left(\frac{1}{z}\right).$$

Prove that  $f(z)$  is constant.

**Problem 5.** (10 points) Calculate the positively oriented contour integral

$$\int_{|z|=1} \frac{e^{3z}}{z^2 - \frac{1}{9}} dz.$$

**Problem 6.** (10 points) Calculate the positively oriented contour integral

$$\int_{|z|=1000} \frac{z^{16} e^{\frac{1}{2z}}}{(z^9 - 625)(z^8 - 346)} dz.$$

**Problem 7.** (15 points) Calculate the integral

$$\int_0^\infty \frac{\sqrt[4]{x}}{x^2 + 16} dx.$$

Make sure to show all your work.

**Problem 8.** (15 points) Calculate the integral

$$\int_0^\infty \frac{\cos(x) - 1}{x^2} dx.$$

Make sure to show all your work.

**Problem 9.** (10 points) Find the number of zeros (counting with multiplicity) of the polynomial

$$2z^{10} + 3z^7 - 9z^2 + 1$$

in the annulus  $1 \leq |z| < 2$ .

**Problem 10.** (10 points) Find a linear fractional transformation that takes the points  $1, i, -i$  to the points  $0, 1, -1$ , respectively.