MATH 417 HOMEWORK 3

You may collaborate on the homework. However, the final write-up must be yours and should reflect your own understanding of the problem. Please be sure to properly cite any help you get.

Problem 1 Consider the composition $f(w(z))$ of two complex valued functions of a complex variable, $f(w)$ and $w(z)$, where $z = x + iy$ and $w = u + iv$. Assume that both functions have continuous partial derivatives. Show that the chain rule can be written in complex form as

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial w}\frac{\partial w}{\partial z} + \frac{\partial f}{\partial \overline{w}}\frac{\partial \overline{w}}{\partial z} \quad \text{and} \quad \frac{\partial f}{\partial \overline{z}} = \frac{\partial f}{\partial w}\frac{\partial w}{\partial \overline{z}} + \frac{\partial f}{\partial \overline{w}}\frac{\partial \overline{w}}{\partial \overline{z}}$$

Show as a consequence that if $f(w)$ is analytic in w and $w(z)$ is analytic in z, then $f(w(z))$ is an analytic function of z.

Problem 2 Let $f(z) = u(x, y) + iv(x, y)$ be an analytic function. Show that

$$|f'(z)|^2 = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}$$

Hence the absolute value of the derivative of an analytic function is the Jacobian of the differentiable transformation of the plane defined by that function.

Problem 3 Can $2x^3 - 6xy^2 + x^2 - y^2 - y$ be the real part of an analytic function? If so, find all possible imaginary parts.

Problem 4 Can $x^2 - y^2 + e^{-y}\sin x - e^{y}\cos x$ be the real part of an analytic function? If so, find all possible imaginary parts.

Problem 5 Determine the minimal conditions on the coefficients a, b, c, d that guarantee that the function $ax^3 + bx^2y + cxy^2 + dy^3$ is harmonic.