

MATH 417: MIDTERM II

This midterm is due Wednesday November 20 in the beginning of class. You may use your class notes, previous homeworks and the course text book. You may not use any other materials, including other text books, the web, question centers, etc. The work should be yours and yours alone. Please do not collaborate. There are 10 problems each worth 10 points.

Problem 1. Find the radius of convergence R of the following power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n z^n}{n}.$$

Discuss the convergence of the series at the boundary points $|z| = R$.

Problem 2. Find all possible Laurent expansions of the function

$$\frac{1}{z(z^2 + 1)}$$

around the point $z = 0$.

Problem 3. For each of the following functions find all the isolated singularities in the complex plane. Determine whether the singularity is removable, a pole or an essential singularity. In case the singularity is a pole, determine its order. Calculate the residue at the singular point.

$$(a) \frac{e^z - z - 1}{z^4} \quad (b) \frac{\sin(z^3) - z^3}{z^6} \quad (c) \frac{e^z}{(z-1)^3}$$

Problem 4. For each of the following functions find all the zeros of the function. Determine the order of the zero at each of these points.

$$(a) (e^z - 1) \cos(z) \quad (b) \frac{\cos(z) - 1}{z} \quad (c) (z^3 + 1)e^z$$

Problem 5. Calculate the integral

$$\int_0^{\infty} \frac{x^2}{x^8 + 1} dx.$$

In order to receive credit you must show all your work.

Problem 6. Let $a > 0, b > 0$ be positive real numbers. Calculate the integral

$$\int_0^{\infty} \frac{\cos(2ax) - \cos(2bx)}{x^2} dx.$$

In order to receive credit you must show all your work.

Problem 7. Calculate the integral

$$\int_0^{\infty} \frac{\ln(x)}{x^4 + 1} dx.$$

In order to receive credit you must show all your work.

Problem 8. Let $0 < \lambda < 1$. Calculate the integral

$$\int_0^\infty \frac{x^\lambda}{(1+x)^2} dx.$$

In order to receive credit you must show all your work.

Problem 9. Calculate the integral

$$\int_0^\infty \frac{\sqrt{x}(\cos(x) + \sin(x))}{x^2 + 1} dx.$$

In order to receive credit you must show all your work.

Problem 10. Calculate the positively oriented contour integral

$$\int_{|z|=3} \frac{z^8 e^{\frac{1}{z}}}{(z^3 + 1)(z^6 + 2)} dz.$$