MATH 417 MIDTERM 1

This midterm is due Wednesday March 5 in the beginning of class. You may use your class notes and the course text book. You may not use any other materials, including other text books, the web, question centers, etc. The work should be yours and yours alone. Please do not collaborate. There are 10 problems each worth 10 points.

Problem 1 Find all solutions of the equation $z^8 = -1 + \sqrt{3}i$. Calculate all the values of $(-1+\sqrt{3}i)^{1/8}$ using the definition of roots via logarithms. Show that your two answers are the same.

Problem 2 Find the principal values of the following expressions.
(1) $(2-2i)^{(1+i)}$
(2) $(2-2\sqrt{3}i)^{(3+4i)}$

Problem 3 Determine whether the function $e^{-x}\cos y + x^4 - 6x^2y^2 + y^4$ can be the real part of an analytic function. If so, find all analytic functions that have it as their real parts.

Problem 4 Let $f(z) = u(x,y) + iv(x,y)$ be an analytic function in a domain D. Let a, b be two real numbers. Assume that $z_0 = x_0 + iy_0$ is on the level curves $u(x,y) = a$ and $v(x,y) = b$ and that $f'(z_0) \neq 0$. Show that the tangent lines to the curves $u(x,y) = a$ and $v(x,y) = b$ at z_0 are perpendicular. (Hint: Show that the dot product of the gradient vectors are zero.)

Problem 5 Calculate the integral

$$\int_C (e^{z^2} + \overline{z}^2)dz$$

where C is the positively oriented boundary of a rectangle with vertices $-1-i, 2-i, 2+i, -1+i$.

Problem 6 Estimate the integral

$$\int_{C_R} \frac{2z^2+7}{z^6+2z^3+1}dz,$$

where C_R is the circle $|z| = R$ (assume $R >> 0$) positively oriented. Show that the integral tends to zero as R tends to infinity.

Problem 7 Calculate the integral

$$\int_C \frac{e^z\cos(z)}{z^3}dz,$$

where C is the unit circle taken with the positive orientation

Problem 8 Calculate the integral

$$\int_C \frac{z^5 + 3z^2 - 7}{(z - \frac{1}{2})(z + \frac{1}{2})} dz,$$

where C is the unit circle taken with the positive orientation

Problem 9 Assume that f is an analytic function in a domain D that takes only real values. Prove that that f has to be constant.

Problem 10 Suppose that f is a non-constant analytic function in a closed, bounded region R. Suppose $f(z) \neq 0$ at any point of R. Show that $|f(z)|$ attains both its minimum and maximum value on the boundary of R and never in the interior of R.