

## MATH 417 HOMEWORK 4

You may collaborate on the homework. However, the final write-up must be yours and should reflect your own understanding of the problem. Please be sure to properly cite any help you get.

**Problem 1** Prove the following formulae involving trigonometric functions

- (1)  $2 \sin(z_1) \cos(z_2) = \sin(z_1 + z_2) + \sin(z_1 - z_2)$
- (2)  $\cos(z_1 + z_2) = \cos(z_1) \cos(z_2) - \sin(z_1) \sin(z_2)$
- (3)  $|\cos(z)|^2 = \cos^2(x) + \sinh^2(y)$ , where  $z = x + iy$ .

**Problem 2** Show that

- (1)  $|\sinh(y)| \leq |\sin(z)| \leq \cosh(y)$ ; and
- (2)  $|\sinh(y)| \leq |\cos(z)| \leq \cosh(y)$ , where  $z = x + iy$ .

**Problem 3** Calculate  $\text{Log}(1 + i)$  and  $\text{Log}(-ei)$ .

**Problem 4** Find the principal values of  $i^{2i}$  and  $(1 + i)^{4i}$ .

**Problem 5** Find all the values of  $(1 + \sqrt{3}i)^{3/2}$ .

**Extra Credit:** Find the genus of the following Riemann surfaces

- (1)  $y^2 = x(x^2 - 1)(x^2 - 4)$
- (2)  $y^2 = (x^2 - 1)(x^2 - 4)(x^2 - 9)(x^2 - 16)$