

1. MATH 494 HOMEWORK 2

This homework is due Wednesday January 29 in the beginning of class. No late homework will be accepted. You may collaborate on the homework. However, the final write-up must be yours and should reflect your own understanding of the problem. Please be sure to properly cite any help you get.

Problem 1.1. Find generators of the ideal $I(V)$, where V is the following sets of points in $\mathbb{A}_{\mathbb{C}}^2$

(1) $V = \{(0, 0), (1, 1), (0, 1), (1, 0)\}$.

(2) $V = \{(0, 0), (1, 1), (2, 2), (0, 1)\}$.

(3) $V = \{(0, 0), (1, 1), (2, 2), (-1, -1)\}$

How do the sets of generators differ in these examples? Is there a geometric explanation for this difference?

Problem 1.2. Consider the image of the map $f : \mathbb{A}_{\mathbb{C}}^1 \rightarrow \mathbb{A}_{\mathbb{C}}^3$ given by $f(t) = (t, t^2, t^3)$. Find generators of the ideal of the image.

Problem 1.3. Consider the image of the map $f : \mathbb{A}_{\mathbb{C}}^1 \rightarrow \mathbb{A}_{\mathbb{C}}^3$ given by $f(t) = (t^3, t^4, t^5)$. Find generators of the ideal of the image. How many generators did you need? Can you generate the ideal with fewer polynomials?

Problem 1.4. Let $l_1 = \{x = y = 0\}$, $l_2 = \{x - y = z = 0\}$ and $l_3 = \{x - y - 1 = z = 0\}$. Find generators of the ideal $I(l_1 \cup l_2)$ and $I(l_1 \cup l_3)$. How do these cases differ?

Problem 1.5. Let V be the union of the three coordinate axes (x-axis ($y = z = 0$), y-axis ($x = z = 0$), z-axis ($x = y = 0$)) in $\mathbb{A}_{\mathbb{C}}^3$. Find generators of $I(V)$.