1. MATH 494: Homework 4

This problem set is due Friday October 8. You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding.

Problem 1.1. Show that any two lines in $\mathbb{P}^2_k$ intersect. If you take two parallel lines

$$ax + by - c = 0 \quad \text{and} \quad ax + by - d = 0$$

in $\mathbb{R}^2$ (viewed as a distinguished affine in $\mathbb{P}^2_{\mathbb{R}}$), at which point of $\mathbb{P}^2_{\mathbb{R}}$ do they intersect?

Problem 1.2. Let C be an irreducible conic in $\mathbb{P}^2_{\mathbb{C}}$. Show that C intersects every curve defined by a homogeneous polynomial F of degree d and not vanishing identically on C in $2d$ points (counting with multiplicity).

Problem 1.3. Let Λ be an s-dimensional linear space in $\mathbb{P}^n_k$. Let Γ be a t-dimensional linear space in $\mathbb{P}^n_k$. Show that Λ and Γ have a non-empty intersection if $s + t \geq n$.

Problem 1.4. Using the previous problem, show that given a set of $n \leq d(d+3)/2$ points in $\mathbb{P}^2$, there exists a non-zero homogeneous polynomial of degree d in three variables vanishing at all the points.

Problem 1.5. Let me know your final paper topic.