

HOMEWORK 5

This problem set is due Wednesday October 20. You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding.

Problem 0.1. Let $E : y^2 = x^3 + ax + b$ be a non-singular cubic curve. If $P \in E$ has coordinates (x, y) . Calculate the x -coordinate of $2P$. Find the points of order two on a non-singular cubic curve in Weierstrass normal form.

Problem 0.2. Describe the points of order three on a non-singular curve in Weierstrass normal form. Deduce that a non-singular cubic has nine flex lines. Show that the line joining any two inflection points intersects the curve in a third inflection point. Hence, the inflection points of a non-singular cubic are nine non-collinear points such that a line joining any two contains a third.

Problem 0.3. Show that if S is a finite set of non-collinear points in \mathbb{R}^2 , then there is a line containing only two of the points of S . Using the previous exercise, deduce that not all the inflection points on a cubic curve can be real.

Problem 0.4. Find the order of the point $(2, 3)$ on $y^2 = x^3 + 1$.

Problem 0.5. Show that the equation $y^2 = x^3 - 2$ has infinitely many rational solutions.