MATH 417 HOMEWORK 5

You may collaborate on the homework. However, the final write-up must be yours and should reflect your own understanding of the problem. Please be sure to properly cite any help you get.

Problem 1 Let n and m be two integers. Show that

$$\int_0^{2\pi} e^{im\theta} e^{-in\theta} d\theta = 0$$

if $n \neq m$ and is 2π is $n = m$.

Problem 2 Evaluate the following integrals:

(1) The integral

$$\int_C \frac{z+4}{z} dz$$

for each of the following contours

(a) The semi-circle $z = 4e^{i\theta}$ for $0 \leq \theta \leq \pi$

(b) The circle $z = 4e^{i\theta}$ for $0 \leq \theta \leq 2\pi$

(2) The integral

$$\int_C \pi \exp(\pi \overline{z}) dz$$

where C is the boundary of the square with vertices $0, 1, 1+i, i$ oriented in the counterclockwise direction starting and ending at 0.

Problem 3 Let C denote the line segment from $z = i$ to $z = 1$. Show that

$$\left| \int_C \frac{dz}{z^4} \right| \leq 4\sqrt{2}.$$

Problem 4 Let C_R denote the upper half circle $|z| = R$ (for $R > 2$) parameterized in the counterclockwise direction. Show that

$$\left| \int_{C_R} \frac{2z^2 - 1}{z^4 + 5z^2 + 4} dz \right| \leq \frac{\pi R(2R^2 + 1)}{(R^2 - 1)(R^2 - 4)}$$

Conclude that the integral tends to zero as R tends to infinity.

Problem 5 Let C_R denote the upper half circle $|z| = R$ (for $R > 1$) parameterized in the counterclockwise direction. Show that

$$\left| \int_{C_R} \frac{Log(z)}{z^2} dz \right| \leq 2\pi \frac{\pi + \ln R}{R}$$

Conclude that the integral tends to zero as R tends to infinity.