This problem set is due Friday October 30. You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding. In all these exercises assume that $k$ is an algebraically closed field and $R$ is a commutative ring with unit.

**Problem 0.1.** Calculate the Hilbert polynomial of a linear space of dimension $k$ in $\mathbb{P}^n$.

**Problem 0.2.** Calculate the Hilbert function of the rational normal curve of degree $d$ given by

$$[x_0, x_1] \rightarrow [x_0^d, x_0^{d-1}x_1, \ldots, x_1^d]$$

by noting that the homogeneous polynomials of degree $m$ in the coordinates of $\mathbb{P}^d$ pull-back to give all homogeneous polynomials of degree $md$ in two variables. More generally, using the same observation show that the Hilbert function of the $d$-th Veronese image of $\mathbb{P}^n$ is given by

$$h(m) = p(m) = \binom{md + n}{n}.$$ 

**Problem 0.3.** Calculate the Hilbert polynomial of a hypersurface of degree $d$ in $\mathbb{P}^n$.

**Problem 0.4.** Calculate the Hilbert polynomial of a pair of skew lines in $\mathbb{P}^3$. Calculate the Hilbert polynomial of a pair of intersecting lines in $\mathbb{P}^3$.

**Problem 0.5.** Calculate the Hilbert polynomial of three concurrent lines in $\mathbb{P}^3$ that do not lie in a plane. Calculate the Hilbert polynomial of three concurrent lines in $\mathbb{P}^3$ that do lie in a plane. Are these closed algebraic sets isomorphic?