

HOMEWORK 6

This problem set is due Friday October 30. You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding. In all these exercises assume that k is an algebraically closed field and R is a commutative ring with unit.

Problem 0.1. Calculate the Hilbert polynomial of a linear space of dimension k in \mathbb{P}^n .

Problem 0.2. Calculate the Hilbert function of the rational normal curve of degree d given by

$$[x_0, x_1] \rightarrow [x_0^d, x_0^{d-1}x_1, \dots, x_1^d]$$

by noting that the homogeneous polynomials of degree m in the coordinates of \mathbb{P}^d pull-back to give all homogeneous polynomials of degree md in two variables. More generally, using the same observation show that the Hilbert function of the d -th Veronese image of \mathbb{P}^n is given by

$$h(m) = p(m) = \binom{md + n}{n}.$$

Problem 0.3. Calculate the Hilbert polynomial of a hypersurface of degree d in \mathbb{P}^n .

Problem 0.4. Calculate the Hilbert polynomial of a pair of skew lines in \mathbb{P}^3 . Calculate the Hilbert polynomial of a pair of intersecting lines in \mathbb{P}^3 .

Problem 0.5. Calculate the Hilbert polynomial of three concurrent lines in \mathbb{P}^3 that do not lie in a plane. Calculate the Hilbert polynomial of three concurrent lines in \mathbb{P}^3 that do lie in a plane. Are these closed algebraic sets isomorphic?