## MATH 553: FINAL PROBLEM SET

The following problems are due Wednesday April 28, 2021. You may work in groups, but the final write-up must be your own work.
Problem 1. Show that the Hilbert scheme of $n$ points on $\mathbb{P}^{1}$ is isomorphic to $\mathbb{P}^{n}$
Problem 2. Show that the Hilbert scheme $\operatorname{Hilb}_{2 t+1}\left(\mathbb{P}^{3}\right)$ is smooth and irreducible of dimension 8. Describe the geometry of $\operatorname{Hilb}_{2 t+1}\left(\mathbb{P}^{3}\right)$ as explicitly as you can.
Problem 3. Show that the Hilbert scheme of twisted cubics $\operatorname{Hilb}_{3 t+1}\left(\mathbb{P}^{3}\right)$ has at least 2 components. The general point of one component parameterizes twisted cubics. The general point of the other component parameterizes a plane cubic curve union a point. Compute the dimension of each of these components. Where do these components intersect? If you are feeling energetic, show that these are the only two components.

Problem 4. Show that the Hilbert scheme $\operatorname{Hilb}_{3 t}\left(\mathbb{P}^{3}\right)$ is smooth and irreducible of dimension 12. Describe the geometry of $\operatorname{Hilb}_{3 t}\left(\mathbb{P}^{3}\right)$ as explicitly as you can.

Problem 5. Prove that $\operatorname{Hilb}_{96}\left(\mathbb{C}^{3}\right)$ is reducible. (Hint: Consider ideals $I=\left(V, m^{8}\right)$, where $m$ is the maximal ideal at the origin and $V$ is a 24 -dimensional vector space of homogeneous polynomials of degree 7)
Problem 6. Prove that $\operatorname{Hilb}_{8}\left(\mathbb{C}^{4}\right)$ is reducible. (Hint: Consider ideals $I=\left(V, m^{3}\right)$, where $m$ is the maximal ideal at the origin and $V$ is a 7 -dimensional vector space of homogeneous polynomials. Check that the Hilbert scheme is smooth at such a point if $V$ is spanned by $x_{1}^{2}, x_{1} x_{2}, x_{2}^{2}, x_{3}^{2}, x_{3} x_{4}, x_{4}^{2}, x_{1} x_{4}+x_{2} x_{3}$.)

