

MATH 417 HOMEWORK 8

You may collaborate on the homework. However, the final write-up must be yours and should reflect your own understanding of the problem. Please be sure to properly cite any help you get.

Problem 1 Find the Taylor series expansion of $\text{Log}(1+z)$ around $z = 0$. Determine the radius of convergence of this series. Using your expression find the value of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}.$$

Problem 2 Derive the Laurent series expansion of

$$\frac{e^z}{(z+1)^2}$$

around $z = -1$.

Problem 3 By differentiating the Taylor series expansion for the geometric series, find the Taylor expansion of the following functions around $z = 0$

$$(a) \frac{1}{(1-z)^2} \quad (b) \frac{1}{(1-z)^3} \quad (c) \frac{1}{(1-z)^k}$$

Problem 4 Expand the function

$$\frac{1}{z(z-1)(z-2)}$$

in Laurent series in z in all possible regions.

Problem 5 Does there exist a function $f(z)$ analytic in a neighborhood of the origin such that $|f^{(n)}(0)| \geq n^n$? Explain your answer. What is the largest disc around the origin in which such a function can be analytic? Does there exist a function $f(z)$ analytic in a neighborhood of the origin such that $|f^{(n)}(0)| \geq n!n^n$? Explain your answer.