MATH 417 HOMEWORK 9

You may collaborate on the homework. However, the final write-up must be yours and should reflect your own understanding of the problem. Please be sure to properly cite any help you get.

Problem 1 For each of the following functions find all the isolated singularities. Determine whether they are removable singularities, poles or essential singularities. If the function has a pole, determine the order of the pole.

$$(a)\ \frac{\sin(z^2)}{z^2} \quad (b)\ \frac{\cos(z)-1}{z^4} \quad (c)\ (\tan z)^2 \quad (d)\ \tan(z^2)$$

Problem 2 Find the residues of $f(z)dz$ at all the isolated singular points in the complex plane for the following functions $f(z)$

$$(a)\ \frac{z+5}{z(z^2+1)} \quad (b)\ \frac{2z+1}{(z-1)^2(z+1)} \quad (c)\ z^2e^{1/z}$$

Problem 3 Let C be the positively oriented simple closed contour $|z|=3$. Calculate the following integrals.

$$(a)\ \int_C \frac{ze^z}{z^2-1}dz \quad (b)\ \int_C \frac{\cosh(\pi z)}{z(z^2+1)}dz \quad (c)\ \int_C \frac{z^3e^{1/z}}{1+z^3}dz$$

Problem 4 For the following two problems you must justify your answer.

(1) Find all functions $f(z)$ that are analytic in the entire complex plane and satisfy $2|\sin(z)| \geq |f(z)|$.

(2) Find all functions $f(z)$ that are analytic in the entire complex plane and satisfy $2|f(z)| \geq |\sin(z)|$.

Problem 5 For each of the following functions find the order of zero at $z=0$.

$$(a)\ \cos(z^2)-1 \quad (b)\ \sin(z^3) \quad (c)\ (e^z-1)^3 \quad (d)\ z^7-12z^5+z^3$$