1. For those who enjoy thinking about the moduli space of curves.

Here are some basic questions that one can ask about the moduli space of curves $\overline{M}_g$ and some papers that address these questions. The references that I suggest are by no means complete, the most recent or even the most accessible. It is just a list of fun papers to get started.

Question 1: What is the cohomology of $\overline{M}_g$? A complete answer is currently far from accessible, but a lot has been written on the subject. See [AC], [Dia], [Har1], [Har2], [HZ]. If the cohomology is too hard, how about the tautological ring? See [Fab], [FaP1], [FaP2]. See also recent work of Graber and Vakil.
Question 2: What is the Kodaira dimension of $\overline{M}_g$? See [EH5], [H], [HM2]. There is some recent work of Gabi Farkas extending the work of Eisenbud, Harris, Mumford and Adam Logan. See his web page.

Question 3: What are the ample divisors on $\overline{M}_g$? What are the effective divisors on $\overline{M}_g$? See [HMo], [GKM], [FaP3]. Deepee Khosla and Gabi Farkas have some fascinating new work on the subject.

Question 4: What is Brill-Noether theory? How can it be used to understand the geometry of curves? See [HM1], [EH3], [EH6], [EH4] , [EH2], [EH1].

Question 5: What is the moduli space of curves? How does one construct it? See [HM1], [MFK].

2. For those who enjoy thinking about Kontsevich moduli spaces and quantum cohomology.

Question 1. How do I start learning about stable maps and quantum cohomology? See [FP].

Question 2. What is the cohomology of the Kontsevich moduli space? What is the divisor theory of the Kontsevich moduli space? See [Pa1], [Pa2]. There is some recent work on the Chow rings of the Kontsevich moduli spaces by Anca and Andrei Mustata. See Intermediate Moduli Spaces of Stable Maps, math.AG/0409569 and On the Chow ring of $\overline{M}_{0,m}(n,d)$, math.AG/0507464.

Question 3. How does one compute Gromov-Witten invariants? See [Ga2], [Ga1], [V], [GP].

Question 4. Can the moduli space of stable maps be used to give cool applications to questions about vector bundles? See [PR].

Question 5. What other cool things are going on in the subject? Some more recent work can be found on the following preprints by R. Pandharipande, A. Okunkov, D. Maulik, J. Bryan and collaborators listed below.

(1) A topological view of Gromov-Witten theory, math.AG/0412503
(2) Quantum cohomology of the Hilbert scheme of points in the plane, math.AG/0411210
(3) The local Gromov-Witten theory of curves, math.AG/0411037
(4) Gromov-Witten theory and Donaldson-Thomas theory, I, math.AG/0312059
(5) Gromov-Witten theory and Donaldson-Thomas theory, II, math.AG/0406092
(6) The equivariant Gromov-Witten theory of $\mathbb{P}^1$, math.AG/0207233

3. For those who find dimension 1 claustrophobic.

How about the moduli spaces of surfaces? Here even existence is very recent and almost nothing is known about their detailed geometry. See however [KSB], [Al1], [Al2], [BK].

One can consider many other moduli problems such as moduli spaces of vector bundles on a curve or surface, the moduli space of abelian varieties, moduli spaces of polarized K3 surfaces, moduli spaces of pairs, etc. There has been a lot of progress in questions relating to many of these moduli spaces in recent years. See the work of Alexeev, Paul Hacking, Klaus Hulek among many others.

4. For those who like Mori theory and birational geometry.

For a great introduction to the subject see the two books [Deb], [KM]. If you are interested, you can explore the papers of Kollár, Miyaoka, Mori, Reid, Shokurov and others on the subject.

Recently there has been some new developments in the field. For example, see the preprints of Hacon and McKernan.

(1) On the existence of Flips math.AG/0507597
(2) Boundedness of pluricanonical maps of varieties of general type math.AG/0504327
(3) Shokurov's Rational Connectedness Conjecture math.AG/0504330

There is also some recent work of Boucksom, Campana, Demailly, Paun, Peternell and collaborators that would be great to explore. See for example, The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension math.AG/0405285.

5. For those who like the proper mix of birational geometry and moduli theory.

Brendan Hassett has been doing some fascinating work combining Mori theory and the moduli space of curves. It would be good to explore his work. Another great paper to explore would be [Th].

6. For those who enjoy questions of rational connectivity and finding sections.

Rational connectivity has emerged in the last decade as a very useful substitute for rationality and unirationality in studying the geometry varieties. Rationally connected varieties exhibit interesting properties. In the past decade we have learned a lot about them, but many questions remain. Some good papers and books to explore would be [GHS], [KSC], [K]. There has been recent progress on the question of weak approximation; see the work of Brendan Hassett and Yuri Tschinkel.

7. For those who enjoy questions of hyperbolicity.

The Mordell-Faltings theorem asserts that a curve of $g \geq 2$ defined over a number field has only finitely many points over any finite field extension of that number field. Hyperbolicity is a way of extending the condition of $g \geq 2$ to higher dimensional varieties. For those who wish to explore the topic see [La], [Dem], [Siu1], [Siu2], [SY], [CHM],

8. For those who prefer questions with more arithmetic flavor.

Under what conditions can one find rational points on a variety? Rational points on an elliptic curve are potentially dense. How about for the higher dimensional analogues of elliptic curves, for example K3 surfaces? Are rational points on K3 surfaces potentially dense? No one knows, but there are many instances when one can prove results about rational points on K3 surfaces. See for example, [BT], [HT], [Br]. It would be fun to explore recent work of Ronald van Luijk, Brendan Hassett and Yuri Tschinkel on the subject.

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