This problem set is due Friday November 20. You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding.

**Problem 0.1.** Let $E : y^2 = x^3 + ax + b$ be a non-singular cubic curve. If $P \in E$ has coordinates $(x, y)$. Calculate the $x$-coordinate of $2P$. Find the points of order two on a non-singular cubic curve in Weierstrass normal form.

**Problem 0.2.** Describe the points of order three on a non-singular curve in Weierstrass normal form. Deduce that a non-singular cubic has nine flex lines. Show that the line joining any two inflection points intersects the curve in a third inflection point. Hence, the inflection points of a non-singular cubic are nine non-collinear points such that a line joining any two contains a third. Is it possible to have a set $S$ of finitely many non-collinear real points in $\mathbb{R}^2$ such that the line joining any two contains another point of $S$?

**Problem 0.3.** Let $E : y^2 = x^3 + ax + b$ be a non-singular cubic with $a$ and $b$ in a number field $k$. Prove that the $k$-points $E(k)$ form a subgroup of $E(\mathbb{C})$.

**Problem 0.4.** Show that the equation $y^2 = x^3 - 2$ has infinitely many rational solutions.