MATH 417 MIDTERM 1

This midterm is due Wednesday October 15 in the beginning of class. You may use your class notes and the course text book. You may not use any other materials, including other text books, the web, question centers, etc. The work should be yours and yours alone. Please do not collaborate. There are 10 problems each worth 10 points.

Problem 1 Find all solutions of the equation $z^6 = 1 - i$. Calculate all the values of $(1-i)^{1/6}$ using the definition of roots via logarithms. Show that your two answers are the same.

Problem 2 Find all the values of the following expressions. Indicate their principal values.

(1) $\log(e + ei)$
(2) $(2i)^{4i}$

Problem 3 Determine whether the function $e^x \sin y + x^5 - 10x^3y^2 + 5xy^4$ can be the real part of an analytic function. If so, find all analytic functions that have it as their real parts.

Problem 4 Let $f(z)$ be an entire function. Let $u(x, y)$ be the real part of f. Suppose $u(x, y) \leq M$ for some positive constant M for every $z = x + iy$. Prove that $f(z)$ is constant.

Problem 5 Calculate the integral

$$\int_C (e^{cos(z)} - z^2 + \overline{z}^2) dz$$

where C is the positively oriented boundary of a rectangle with vertices $-2 - i, 1 - i, 1 + i, -2 + i$.

Problem 6 Estimate the integral

$$\int_{C_R} \frac{z^4 + 2}{z^6 - 3z^4 + 3z^2 - 1} dz,$$

where C_R is the circle $|z| = R$ (assume $R >> 0$) positively oriented. Show that the integral tends to zero as R tends to infinity.

Problem 7 Calculate the integral

$$\int_C \frac{e^{z^2} \sin(z)}{z^2} dz,$$

where C is the rectangle with vertices $5 + 5i, -5 + 5i, -5 - 5i, 5 - 5i$ taken with the positive orientation.

Problem 8 Calculate the integral

$$\int_C \frac{z^5 + 2z^2 - 5}{(z-1)(z+1)} dz,$$

where C is the circle of radius three centered at the origin taken with the positive orientation

Problem 9 Let f be an analytic function in a domain D. Suppose that the image of f is contained in the line $ax+by=c$, where $z=x+iy$. Prove that f is constant.

Problem 10 Let f be an entire function. Suppose that $|f(z)| \leq |z|^n$ when $|z| > R$. Prove that $f(z)$ is a polynomial of degree at most n.