MATH 552 ALGEBRAIC GEOMETRY I

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Welcome to Math 552! This course serves as an introduction to Algebraic Geometry. Algebraic Geometry is a central subject in modern mathematics, with close connections with number theory, combinatorics, representation theory, differential and symplectic geometry. We will study basic properties of projective algebraic varieties such as dimension, degree and singularities. At the same time, we will develop a large body of examples that motivate the study of the subject. Depending on time, we will develop the classical theory of curves and surfaces. This course should be enough preparation for a course on the theory of schemes.

Course webpage: http://www.math.uic.edu/~coskun/math5522015.html

Venue: Taft Hall 304

Office hours: M 10-12, W 10-11 and by appointment in SEO 423.

Text: There are three recommended texts for this course.

- (FC) Joe Harris, Algebraic Geometry: a first course, Springer 1992.
- (BAG) Igor Shafarevich, Basic Algebraic Geometry I, Varieties in Projective Space, Springer-Verlag 1994.
- David Mumford, Algebraic Geometry I, Complex Projective Varieties, Springer 1995.

Prerequisites: A solid background in commutative algebra, especially in the theory of rings and modules at the level of a first year graduate class. Some familiarity with complex analysis, algebraic topology and differential geometry useful, but not required.

Requirements: There will be weekly homework. Homework is a very important component of this course. It will count for 100 % of your grade. No late homework will be accepted. You may collaborate on the homework problems, but you must write your own solutions and properly acknowledge any help you receive from others.

Topics: The following is a tentative list of topics that will be covered in the course. Please read the subject in the recommended texts before class.

| Aug 24 | Affine varieties | BAG p. 22-32 |
|--------|-------------------------------------|----------------|
| Aug 26 | Examples: Plane curves | BAG p.1-21 |
| Aug 28 | Rational Functions | BAG p. 32-40 |
| Aug 31 | Projective varieties | BAG p. 41-53 |
| Sep 2 | Examples | FC p. 1-16 |
| Sep 4 | Products | BAG p. 54-60 |
| Sep 7 | Labor Day | No Class |
| Sep 9 | Maps of projective varieties | FC p. 17-31 |
| Sep 11 | Examples; Finite maps | BAG p. 61-66 |
| Sep 14 | Dimension | BAG p. 67-76 |
| Sep 16 | Dimension | FC p. 133-150 |
| Sep 18 | Dimension of Fibers | BAG p. 76-82 |
| Sep 21 | Examples: Grassmannians | FC p. 63-71 |
| Sep 23 | Applications | FC p. 151-162 |
| Sep 25 | Hilbert Polynomials | FC p. 163-173 |
| Sep 28 | Degree of projective varieties | FC p. 88-97 |
| Sep 30 | Degree | FC p. 224-238 |
| Oct 2 | Examples | FC p. 239-250 |
| Oct 5 | Tangent spaces | BAG p. 83-97 |
| Oct 7 | Examples | FC p. 174-185 |
| Oct 9 | Gauss maps, dual varieties | FC p. 186-199 |
| Oct 12 | Power series rings | BAG p. 98-113 |
| Oct 14 | Blow-ups | BAG p. 114-124 |
| Oct 16 | Blow-ups | FC p. 72-87 |
| Oct 19 | Normal varieties | BAG p. 125-131 |
| Oct 21 | Curve singularities | BAG p. 131-138 |
| Oct 23 | Divisors | BAG p. 151-159 |
| Oct 26 | Divisors | BAG p. 159-166 |
| Oct 28 | Divisors on curves | BAG p. 168-174 |
| Oct 30 | Rational curves | |
| Nov 2 | Elliptic curves | BAG p. 175-187 |
| Nov 4 | The group law on elliptic curves | |
| Nov 6 | Abelian varieties | BAG p. 188-194 |
| Nov 9 | Differential forms | BAG p. 195-204 |
| Nov 11 | Differential forms | BAG p. 204-210 |
| Nov 13 | The canonical class | BAG p. 210-215 |
| Nov 16 | The Riemann-Roch Theorem for curves | BAG p. 215-222 |
| Nov 18 | Applications | |
| Nov 20 | Intersection numbers | BAG p. 223-232 |
| Nov 23 | Intersection numbers | BAG p. 232-236 |
| Nov 25 | Bezout's theorem | BAG p. 236-241 |
| Nov 27 | No class: Thanksgiving | |
| Nov 30 | Surfaces | BAG p. 241-245 |
| Dec 1 | Birational maps between surfaces | BAG p. 251-261 |
| Dec 3 | Cubic surfaces | BAG p. 246-251 |