Stat401 HW#11 Solution

§3.2: 1) $X \sim \text{Poisson}(m)$: $p(x) = \frac{m^x e^{-m}}{x!}$ with $x = 0, 1, 2, \cdots$

$P(X=1) = P(X=2) \Leftrightarrow \frac{m^1 e^{-m}}{1!} = \frac{m^2 e^{-m}}{2!} \Leftrightarrow m = 2$; so $p(X=4) = \frac{m^4 e^{-m}}{4!} = \frac{2^4 e^{-2}}{4!} \approx 0.09$

8) $X \sim \text{Poisson}(m)$.

$P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - \frac{m^0 e^{-m}}{0!} - \frac{m^1 e^{-m}}{1!} = 1 - e^{-m} - me^{-m} = 1 - e^{-m}(1+m) > 0.99$

so $e^{-m}(1+m) < 0.01$; Try several values of m to find the desired value.

$m=5$: $e^{-5}(1+5) = 0.0404 > 0.01$

$m=6$: $e^{-6}(1+6) = 0.0173 > 0.01$

$m=7$: $e^{-7}(1+7) = 0.00729 < 0.01$

since for poisson distribution, mean $\mu = m$.

So the smallest value of mean is 7;

10) $X \sim \text{Poisson}(3)$: $p(x) = \frac{3^x e^{-3}}{x!}$, $x = 0, 1, 2, \cdots$

then we're looking for k s.t. $P(X > k) < 0.01$; that is: $P(X \leq k) > 0.99$

From table I in the book, when $m=3$, $k=8$; $P(X \leq 8) = 0.996 > 0.99$. so $k=8$;

§3.3: 2) $X \sim \chi^2(5)$, so $r=5$. $P(X < c) = 0.025 = P(X \leq c)$

$P(c < X < d) = P(X \leq d) - P(X \leq c) = P(X \leq d) - 0.025 = 0.95 \Rightarrow P(X \leq d) = 0.975$

By table II in the appendix, $c = 0.831$; $d = 12.833$.