Stat401 HW#12 Solution

§3.2: 1) $X \sim \text{Poisson}(m)$: $p(x) = \frac{m^x e^{-m}}{x!}$ with $x = 0, 1, 2, \cdots$

$P(X=1) = P(X=2) \Leftrightarrow \frac{\cancel{m}^1 e^{\cancel{-m}}}{1!} = \frac{m^2 e^{\cancel{-m}}}{2!} \Leftrightarrow m = 2$; so $p(X=4) = \frac{m^4 e^{-m}}{4!} = \frac{2^4 e^{-2}}{4!} \approx 0.09$

8) $X \sim \text{Poisson}(m)$.

$P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - \frac{m^0 e^{-m}}{0!} - \frac{m^1 e^{-m}}{1!} = 1 - e^{-m} - me^{-m} = 1 - e^{-m}(1+m) > 0.99$

so $e^{-m}(1+m) < 0.01$; Try several values of m to find the desired value.

$m=5$: $e^{-5}(1+5) = 0.0404 > 0.01$

$m=6$: $e^{-6}(1+6) = 0.0173 > 0.01$

$m=7$: $e^{-7}(1+7) = 0.00729 < 0.01$

since for poisson distrition, mean $\mu = m$.
So the smallest value of mean is 7;

10) $X \sim \text{Poisson}(3)$: $p(x) = \frac{3^x e^{-3}}{x!}$, $x = 0, 1, 2, \cdots$

then we're looking for K s.t. $P(X > K) \leq 0.01$; that is: $P(X \leq K) > 0.99$

From table I in the book, when $m=3$, $K=8$: $P(X \leq 8) = 0.996 > 0.99$. So $K=8$;

§3.3: 2) $X \sim \chi^2(5)$, so $r=5$. $P(X<c) = 0.025 = P(X \leq c)$

$P(c<X<d) = P(X \leq d) - P(X \leq c) = P(X \leq d) - 0.025 = 0.95 \Rightarrow P(X \leq d) = 0.975$

By table II in the appendix, $c = 0.831$; $d = 12.833$.

§3.4: 2) $X \sim N(75, 100)$: $P(X<60) = P\left(\frac{X-\mu}{\sigma} < \frac{60-75}{10}\right) = P(Z < -1.5) = \Phi(-1.5) = 1 - \Phi(1.5) \overset{\text{table III}}{=\!=} 1 - 0.9332 = 0.0678$

$P(70<X<100) = P\left(\frac{70-75}{10} < \frac{X-\mu}{\sigma} < \frac{100-75}{10}\right) = P(-0.5<Z<2.5) = \Phi(2.5) - \Phi(-0.5)$

$= \Phi(2.5) - [1 - \Phi(0.5)] \overset{\text{Table IV}}{=} 0.9938 - 1 + 0.6915 = 0.6853$

3) $X \sim N(\mu, \sigma^2)$. $P\left(-b < \frac{X-\mu}{\sigma} < b\right) = 0.90$

$P(-b<Z<b) = \Phi(b) - \Phi(-b) = \Phi(b) - [1 - \Phi(b)] = 2\Phi(b) - 1 = 0.90 \Rightarrow \Phi(b) = 0.95 \xRightarrow{\text{table III}} b = 1.645$.

4) $\left.\begin{array}{l} P(X<89) = 0.90 \\ P(X<94) = 0.95 \end{array}\right\} \Rightarrow \left.\begin{array}{l} P\left(Z < \frac{89-\mu}{\sigma}\right) = 0.90 \\ P\left(Z < \frac{94-\mu}{\sigma}\right) = 0.95 \end{array}\right\} \xRightarrow[\text{pnorm}]{\text{table III}} \left.\begin{array}{l} \frac{89-\mu}{\sigma} = 1.282 \\ \frac{94-\mu}{\sigma} = 1.645 \end{array}\right\} \begin{array}{l} \mu = 71.3416 \\ \sigma = 13.774 \end{array}$

§3.5: 1) $X \sim N(2.8, 0.16)$. $Y \sim N(110, 100)$. $\rho = \frac{3}{5}$

(1) $P(106<Y<124) = P\left(\frac{106-110}{10} < Z < \frac{124-110}{10}\right) = P(-0.4<Z<1.4) = \Phi(1.4) - \Phi(-0.4) = \Phi(1.4) - [1 - \Phi(0.4)]$

$\overset{\text{table III}}{=} 0.9192 - 1 + 0.6554 = 0.5746$

(2) The conditional dist of Y given $X=x$ is $N\left[\mu_y + \rho\frac{\sigma_y}{\sigma_x}(x - \mu_x), \sigma_y^2(1-\rho^2)\right]$

$\Rightarrow E(Y|X=3.2) = \mu_y + \rho\frac{\sigma_y}{\sigma_x}(3.2 - \mu_x) = 110 + \frac{3}{5} \cdot \frac{10}{0.4}(3.2 - 2.8) = 116$;

$Var(Y|X=3.2) = \sigma_y^2(1-\rho^2) = 100\left[1 - \left(\frac{3}{5}\right)^2\right] = 64$.

so $P(106<Y<124|X=3.2) = P\left(\frac{106-116}{8} < Z < \frac{124-116}{8}\right) = P(-1.25<Z<1) = \Phi(1) - \Phi(-1.25)$

$= \Phi(1) - [1 - \Phi(1.25)] = 0.8413 - 1 + 0.8944 = 0.7357$.