STAT 401 HW#2 Solution

§1.4 3) $P(C) = \dfrac{\binom{13-4}{1}}{\binom{52-4-1}{1}} = \dfrac{9}{47}$;

5) $C_1 = \{\text{at least 3 kings}\}$; $C_2 = \{\text{at least 2 kings}\}$

$C_1 \cap C_2 = \{\text{at least 3 kings}\} = C_1$

so $$P(C_1|C_2) = \frac{P(C_1 \cap C_2)}{P(C_2)} = \frac{\left[\binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}\right] / \binom{52}{13}}{\left[\binom{4}{2}\binom{48}{11} + \binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}\right] / \binom{52}{13}}$$

$$= \frac{\binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}}{\binom{4}{2}\binom{48}{11} + \binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}}$$

7) $\boxed{a}$ $R_7 = \{\text{Sum of dice is } 7\} = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$R_8 = \{\text{Sum of dice is } 8\} = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$.

So: $\mathbb{P}(R_7) = \frac{6}{36} = \frac{1}{6}$; $\mathbb{P}(R_8) = \frac{5}{36}$;

$$\mathbb{P}(R_7 | R_7 \cup R_8) = \frac{\mathbb{P}(R_7 \cap (R_7 \cup R_8))}{\mathbb{P}(R_7 \cup R_8)} = \frac{\mathbb{P}(R_7)}{\mathbb{P}(R_7) + \mathbb{P}(R_8)} = \frac{1/6}{1/6 + 5/36} = \frac{6}{11}$$

$\boxed{b}$ $R_6 = \{\text{Sum is } 6\} = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$. $\Rightarrow \mathbb{P}(R_6) = \frac{5}{36}$.

So the six and eight occuring before two sevens have combinations:

$(6,8), (8,6), (6,7,8), (8,7,6), (7,6,8), (7,8,6)$

While:

$$\mathbb{P}(R_{6,8}) = \mathbb{P}(R_{8,6}) = \mathbb{P}(R_6 | R_6 \cup R_7 \cup R_8) \cdot \mathbb{P}(R_8 | R_7 \cup R_8)$$

$$= \frac{\mathbb{P}(R_6 \cap (R_6 \cup R_7 \cup R_8))}{\mathbb{P}(R_6 \cup R_7 \cup R_8)} \cdot \frac{\mathbb{P}(R_8 \cap (R_7 \cup R_8))}{\mathbb{P}(R_7 \cup R_8)}$$

$$= \frac{\mathbb{P}(R_6)}{\mathbb{P}(R_6) + \mathbb{P}(R_7) + \mathbb{P}(R_8)} \cdot \frac{\mathbb{P}(R_8)}{\mathbb{P}(R_7) + \mathbb{P}(R_8)}$$

$$= \frac{\frac{5}{36}}{\frac{5}{36} + \frac{1}{6} + \frac{5}{36}} \cdot \frac{\frac{5}{36}}{\frac{1}{6} + \frac{5}{36}}$$

$$= \frac{5}{16} \cdot \frac{5}{11} = \frac{25}{176};$$

$$\mathbb{P}(R_{6,7,8}) = \mathbb{P}(R_{8,7,6}) = \mathbb{P}(R_6 \mid R_6 \cup R_7 \cup R_8)\cdot \mathbb{P}(R_7 \mid R_7 \cup R_8)\cdot \mathbb{P}(R_8 \mid R_7 \cup R_8)$$ page2

$$= \left(\frac{5/36}{5/36 + 1/6 + 5/36}\right)\cdot\left(\frac{1/6}{1/6 + 5/36}\right)\cdot\left(\frac{5/36}{1/6 + 5/36}\right)$$

$$= \frac{5}{16}\cdot\frac{5}{11}\cdot\frac{6}{11} = \frac{150}{1936}$$

$$\mathbb{P}(R_{7,6,8}) = \mathbb{P}(R_{7,8,6}) = \mathbb{P}(R_7 \mid R_6 \cup R_7 \cup R_8)\cdot \mathbb{P}(R_6 \mid R_6 \cup R_7 \cup R_8)\cdot \mathbb{P}(R_8 \mid R_7 \cup R_8)$$

$$= \left(\frac{1/6}{5/36 + 1/6 + 5/36}\right)\cdot\left(\frac{5/36}{5/36 + 1/6 + 5/36}\right)\cdot\left(\frac{5/36}{1/6 + 5/36}\right)$$

$$= \frac{6}{16}\cdot\frac{5}{16}\cdot\frac{5}{11} = \frac{150}{2816}$$

$\Rightarrow \mathbb{P}(\text{six and eight occur before two sevens}) = 2\left(\frac{25}{176} + \frac{150}{1936} + \frac{150}{2816}\right) = \frac{4225}{7744} = 0.546$

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8) Machines I, II, III; Let D denote for defective spring.

then: $P(I) = 0.30$

$P(II) = 0.25$

$P(III) = 0.45$

$P(D \mid I) = 0.01$

$P(D \mid II) = 0.04$

$P(D \mid III) = 0.02$

$\boxed{a}$: $P(D) = P(D \mid I)P(I) + P(D \mid II)P(II) + P(D \mid III)P(III)$

$= 0.01(0.30) + 0.04(0.25) + 0.02(0.45)$

$= 0.022$

$\boxed{b}$ $P(II \mid D) = \dfrac{P(D \mid II)\,P(II)}{P(D)} = \dfrac{0.04(0.25)}{0.022}$

$= \dfrac{5}{11} = 0.4545$

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34) $D = \{\text{Detect Impurity}\}$

$T = \{\text{Impurity exist}\}$

$P(D \mid T) = 0.90$

$P(D \mid T^c) = 0.05$

$P(T) = 0.20$

so: $P(T \mid D) = \dfrac{P(D \mid T)\,P(T)}{P(D)}$

$$= \frac{P(D \mid T)\,P(T)}{P(D \mid T)P(T) + P(D \mid T^c)P(T^c)}$$

$$= \frac{0.90\,(0.20)}{0.90(0.20) + 0.05\,(1 - 0.20)}$$

$$= \frac{0.18}{0.22} = 0.8181$$