§1.7: 1) $\mathcal{C} = \{c: 0 < c < 10\}$. $P(C) = \int_C \frac{1}{10}\, dz$

For $\sqrt{x} < 10$: $F(x) = P(X \le x) = P(c^2 \le x) = P(c \le \sqrt{x}) = \int_0^{\sqrt{x}} \frac{1}{10}\, dz = \frac{1}{10}\sqrt{x}$; ~~for $\sqrt{x} < 10$~~.

For $\sqrt{x} \ge 10$: $F(x) = P(X \le x) = P(C^2 \le x) = P(c < 10) = \int_0^{10} \frac{1}{10}\, dz = 1$;

so cdf of X is $F(x) = \begin{cases} \frac{1}{10}\sqrt{x}, & 0 < x < 100; \\ 1, & \text{elsewhere}; \end{cases}$

then: $f_X(x) = \frac{dF(x)}{dx} = \begin{cases} \frac{d}{dx}\left(\frac{1}{10}\sqrt{x}\right) = \frac{1}{20\sqrt{x}}; & 0 < x < 100. \\ \frac{d}{dx}(1) = 0; & \text{elsewhere}. \end{cases}$

3)

(i) $P_X(C_1 \cup C_2) = P_X(C_1) + P_X(C_2) - P_X(C_1 \cap C_2) = \frac{1}{8} + \frac{1}{2} - 0 = \frac{5}{8}$

(ii) Let $C_3 = \{0 < x \le \frac{1}{4}\}$ then $\mathcal{C} = C_1 \cup C_2 \cup C_3$ and $C_1 \cap C_2 = \phi$. $C_1 \cap C_3 = \phi$. $C_2 \cap C_3 = \phi$

that is: $P_X(C_1 \cup C_2 \cup C_3) = 1$

$\Rightarrow P_X(C_1) + P_X(C_2) + P_X(C_3) = 1 \Rightarrow P_X(C_3) = 1 - \frac{1}{8} - \frac{1}{2} = \frac{3}{8}$

then: $P_X(C_1^c) = P_X(C_2 \cup C_3) = P_X(C_2) + P_X(C_3) = \frac{1}{2} + \frac{3}{8} = \frac{7}{8}$

(iii) $P_X(C_1^c \cap C_2^c) = P_X[(C_1 \cup C_2)^c] = P_X(C_3) = \frac{3}{8}$

6) (a) $f(x) = x^2/18$; $-3 < x < 3$; zero elsewhere.

- $P(|X| < 1) = \int_{-1}^{1} \frac{x^2}{18}\, dx = \frac{1}{18} \cdot \frac{1}{3} x^3 \Big|_{-1}^{1} = \frac{1}{27}$
- $P(\cancel{|X|}\, x^2 < 9) = \int_{-3}^{3} \frac{x^2}{18}\, dx = \frac{1}{18} \cdot \frac{1}{3} x^3 \Big|_{-3}^{3} = \frac{1}{54} \cdot [3^3 - (-3)^3] = 1$

(b) $f(x) = \frac{x+2}{18}$; $-2 < x < 4$; zero elsewhere.

- $P(|X| < 1) = \int_{-1}^{1} \frac{x+2}{18}\, dx = \frac{1}{18} \cdot \left[\frac{1}{2}x^2 + 2x\right]\Big|_{-1}^{1} = \frac{1}{18}\left[\left(\frac{1}{2}+2\right) - \left(\frac{1}{2}-2\right)\right] = \frac{2}{9}$
- $P(x^2 < 9) = P(-3 < x < 3) = P(-2 < x < 3) = \int_{-2}^{3} \frac{x+2}{18}\, dx$

$= \frac{1}{18}\left[\frac{1}{2}x^2 + 2x\right]\Big|_{-2}^{3} = \frac{1}{18}\left[\left(\frac{1}{2}(3)^2 + 2(3)\right) - \left(\frac{1}{2}(-2)^2 + 2(-2)\right)\right] = \frac{25}{36}$

14) $f(x) = 2x$, $0 < x < 1$, zero elsewhere.

$P(X \ge \frac{3}{4} \mid X \ge \frac{1}{2}) = \frac{P(X \ge \frac{3}{4}, X \ge \frac{1}{2})}{P(X \ge \frac{1}{2})} = \frac{P(X \ge \frac{3}{4})}{P(X \ge \frac{1}{2})} = \frac{\int_{\frac{3}{4}}^{1} 2x\, dx}{\int_{\frac{1}{2}}^{1} 2x\, dx} = \frac{x^2\Big|_{\frac{3}{4}}^{1}}{x^2\Big|_{\frac{1}{2}}^{1}} = \frac{1-(\frac{3}{4})^2}{1-(\frac{1}{2})^2} = \frac{7}{12}$

21) $f(x) = 2xe^{-x^2}$; $0 < x < \infty$; zero elsewhere.

$$F_Y(y) = P(Y \le y) = P(X^2 \le y) = P(0 < X \le \sqrt{y})$$
$$= \int_0^{\sqrt{y}} 2xe^{-x^2}\,dx$$

Let $u = -x^2$.
$du = -2x\,dx$
$x = 0.\ u = 0$
$x = \sqrt{y}.\ u = -y$

$$= \int_0^{-y} -e^u\,du$$
$$= -e^u\Big|_0^{-y} = -e^{-y} + 1;\ 0 < y < +\infty.$$

so $f_Y(y) = F_Y'(y) = \begin{cases} e^{-y}; & 0 < y < \infty; \\ 0; & \text{elsewhere}; \end{cases}$

§1.8. 2) $f(x) = \frac{x+2}{18}$; $-2 < x < 4$; zero elsewhere.

- $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-2}^{4} x \cdot \frac{x+2}{18}\,dx = \frac{1}{18}\int_{-2}^{4}(x^2+2x)\,dx = \frac{1}{18}\left(\frac{1}{3}x^3 + x^2\right)\Big|_{-2}^{4} = 2$;
- $E[(X+2)^3] = \int_{-\infty}^{\infty}(x+2)^3 f(x)\,dx = \int_{-2}^{4}(x+2)^3 \cdot \frac{x+2}{18}\,dx = \frac{1}{18}\int_{-2}^{4}(x+2)^4\,dx \;\overset{\substack{u=x+2\\ du=dx}}{\underset{\substack{x=-2,\,u=0\\ x=4,\,u=6}}{=}}\; \frac{1}{18}\int_0^6 u^4\,du$

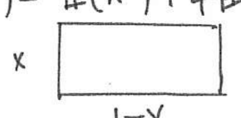
$$= \frac{1}{18}\cdot\frac{1}{5}u^5\Big|_0^6 = \frac{1}{18}\cdot\frac{1}{5}(6^5 - 0^5) = \frac{432}{5};$$

- $E[6X - 2(X+2)^3] = 6E(X) - 2E[(X+2)^3] = 6\cdot 2 - 2\cdot\left(\frac{432}{5}\right) = -\frac{804}{5}$;

3) $p(x) = \frac{1}{5}$; $x = 1, 2, 3, 4, 5$; 0 elsewhere.

- $E(X) = \sum_{i=1}^{5} x\,p(x) = \frac{1}{5}(1+2+3+4+5) = \frac{15}{5} = 3$
- $E(X^2) = \sum_{i=1}^{5} x^2 p(x) = \frac{1}{5}(1^2+2^2+3^2+4^2+5^2) = \frac{1}{5}\cdot 55 = 11$
- $E[(X+2)^2] = E(X^2+4X+4) = E(X^2) + 4E(X) + 4 = 11 + 4(3) + 4 = 27$

6) $f(x) = 3x^2$, $0 < x < 1$



Area $= x(1-x)$

$$E[X(1-X)] = \int_0^1 x(1-x)\cdot(3x^2)\,dx = \int_0^1 (3x^3 - 3x^4)\,dx = \left(\frac{3}{4}x^4 - \frac{3}{5}x^5\right)\Big|_0^1 = \frac{3}{4} - \frac{3}{5} = \frac{3}{20}$$

9) There are $6 \times 5 = 30$ possible outcomes, with equal probability.
Let X represent the absolute difference. then $x = 1, 2, 3, 4, 5$.

$X=x$	1	2	3	4	5
$P(X=x)$	10/30	8/30	6/30	4/30	2/30

10 pairs $= 2 \times \{(1,2), (2,3), (3,4), (4,5), (5,6)\}$ $\quad 2\times\{(1,3), (2,4), (3,5), (4,6)\}$ $\quad 2\times\{(1,4), (2,5), (3,6)\}$ $\quad 2\times\{(1,5), (2,6)\}$ $\quad (1,6), (6,1)$

↑ reverse order.

so $E(X) = \sum_{x=1}^{5} x\,p(x) = 1\cdot\frac{10}{30} + 2\cdot\frac{8}{30} + 3\cdot\frac{6}{30} + 4\cdot\frac{4}{30} + 5\cdot\frac{2}{30}$

$$= \frac{70}{30} = \frac{7}{3};$$