

Problem 1

(a) Plot the distribution function

$$F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x^3 & \text{for } 0 < x < 1 \\ 1 & \text{for } x \geq 1 \end{cases}$$

(b) Determine the corresponding density function $f(x)$ in the three regions(i) $x \leq 0$, (ii) $0 < x < 1$, and (iii) $1 < x$.

(c) What is the mean of the distribution?

Problem 2.

Let Z be a discrete random variable having possible values 0, 1, 2 and 3 and probability mass function

$$p(0) = \frac{1}{4}, \quad p(2) = \frac{1}{8}$$

$$p(1) = \frac{1}{2}, \quad p(3) = \frac{1}{8}$$

(a) plot the corresponding distribution function.

(b) Determine the mean $E(Z)$ (c) Evaluate the variance $\text{Var}(Z)$

Problem 3

Let A, B, C be arbitrary events. Establish the addition law

$$\begin{aligned} P\{A \cup B \cup C\} &= P\{A\} + P\{B\} + P\{C\} \\ &\quad - P\{AB\} - P\{BC\} - P\{AC\} \\ &\quad + P\{ABC\}. \end{aligned}$$

Problem 4

Let X and Y be independent random variables each with the uniform probability density function

$$f(x) = \begin{cases} 1 & 0 < x < 1, \\ 0 & \text{elsewhere.} \end{cases}$$

Find the distribution functions $F_U(x)$ and $F_V(x)$ where $U = \max\{X, Y\}$ and $V = \min\{X, Y\}$.