Stat 461 HW #3

Note Title

Problem 1 The following experiment is performed: An observation is made of a Poisson random variable N with parameter λ. Then N independant Bernoulli Trials are performed, each with probability p of success. Let Z be the total number of successes observed in N trials.

(a) Formulate Z as a random sum and thereby determine its mean and variance.

(b) Give the formula of the probability mass function of Z.

Problem 2. A Markov chain $X_0, X_1, \cdots$ on states $0, 1, 2$ has the following transition probability matrix:

$$
\begin{array}{c c}
 & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\
P = \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0.1 & 0.2 & 0.7 \\ 0.9 & 0.1 & 0 \\ 0.1 & 0.8 & 0.1 \end{pmatrix}
\end{array}
$$

and initial distribution $p_0 = \mathbb{P}\{X_0 = 0\} = 0.3$; $p_1 = \mathbb{P}\{X_0 = 1\} = 0.4$, and $p_2 = \mathbb{P}\{X_0 = 2\} = 0.3$. Determine $\mathbb{P}\{X_0 = 0, X_1 = 1, X_2 = 2\}$.

Problem 3. A Markov chain $X_0, X_1, X_2, \cdots$ has the transition probability matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \end{pmatrix} \end{array}$$

Determine the conditional probability $\mathbb{P}\{X_2 = 1, X_3 = 1 \mid X_1 = 0\}$

Problem 4 Suppose X_n is a Two-state Markov chain whose Transition probability matrix is

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} \alpha & 1-\alpha \\ 1-\beta & \beta \end{pmatrix} \end{array}$$

Then $Z_n = (X_{n-1}, X_n)$ is a Markov chain having the four states $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$. Determine the first row of the Transition probability matrix.