Stat 461 HW #4

Note Title

Problem 1 Consider a spare parts inventory model in which either 0, 1 or 2 repair parts are demanded in any period, with

$$\mathbb{P}\{\xi_n = 0\} = 0.4, \quad \mathbb{P}\{\xi_n = 1\} = 0.3, \quad \mathbb{P}\{\xi_n = 2\} = 0.3,$$

and suppose $s = 0$ and $S = 3$. Determine the transition probability matrix for the Markov chain $\{X_n\}$, where X_n is defined to be the quantity on hand at the end of period n.

Problem 2 Find the mean time to reach 3 starting from state 0 for the Markov chain whose transition probability matrix is

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0 & 0.7 & 0.2 & 0.1 \\ 0 & 0 & 0.9 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Problem 3 Consider the Markov chain whose transition probability matrix is given by

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0.1 & 0.6 & 0.3 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

(a) Starting in state 1, determine the probability that the Markov chain ends in state 0.

(b) Determine the mean time to absorption.

Problem 4 A coin is tossed repeatedly until two successive heads appear. Find the mean number of tosses required.

Hint: Let X_n be the cumulative number of successive heads. The state space is 0, 1, 2 and the transition probability matrix is

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

Determine the mean time to reach state 2 starting from state 0 by invoking a first step analysis.