Stat 461 HW#5

Note Title

Problem 1 Consider a sequence of items from a production process with each item being graded as good or defective. Suppose that a good item is followed by another good item with probability α. Similarly, a defective item is followed by another defective item with probability β. If the first item is good, what is the probability that the second defective item to appear is the fourth item?

Problem 2 An urn initially contains a single red ball and a single green ball. A ball is drawn at random, removed, and replaced by a ball of the opposite color, and this process repeats so that there are always exactly two balls in the urn. Let X_n be the number of red balls in the urn after n draws, with $X_0 = 1$. Specify the transition probabilities for the Markov chain $\{X_n\}$.

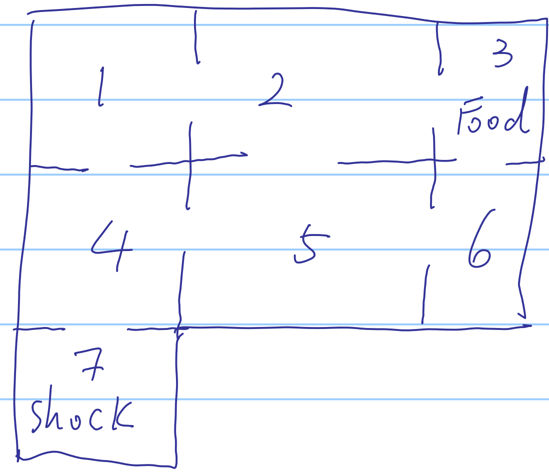
Problem 3 Consider the Markov chain whose transition probability matrix is given by

$$P = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.4 & 0.1 & 0.4 \\ 0.2 & 0.1 & 0.6 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

(a) Starting in state 1, determine the probability that the Markov chain ends in state 0.

(b) Determine the mean time to absorption.

Problem 4 A white rat is put into compartment 4 of the maze shown here:



He moves through the compartment at random; i.e., if there are K ways to leave a compartment, he chooses each of these with probability $1/K$. What is the probability that the rat finds the food in compartment 3 before feeling the electric shock in compartment 7?

Only write down the transition matrix and the equations for the absorption probabilities.