Stat 461 HW #6

Note Title

Problem 1: Consider a general random walk with the following transition matrix

$$P = \begin{array}{c|cccccccc} & 0 & 1 & 2 & 3 & & & \cdots & N \\ \hline 0 & 1 & 0 & 0 & 0 & & & \cdots & 0 \\ 1 & q_1 & r_1 & p_1 & 0 & & & \cdots & 0 \\ 2 & 0 & q_2 & r_2 & p_2 & & & \cdots & 0 \\ 3 & 0 & 0 & q_3 & r_3 & p_3 & & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & & \vdots \\ N & 0 & 0 & 0 & 0 & 0 & & \cdots & 1 \end{array}$$

where $q_k > 0$, $p_k > 0$ for $k = 1, 2, \cdots, N-1$.

Let $T = \min\{n \geq 0 ;\ X_n = 0 \text{ or } X_n = N\}$

and

$$u_i = \mathbb{P}\{X_T = 0 \mid X_0 = i\}$$

By first step analysis, we know that

$$u_i = q_i u_{i-1} + r_i u_i + p_i u_{i+1} \quad \text{for } i = 1, 2, \cdots, N-1$$

and $u_0 = 1$, $u_N = 0$.

Show that:

$$u_i = \frac{\rho_i + \cdots + \rho_{N-1}}{1 + \rho_1 + \rho_2 + \cdots + \rho_{N-1}}, \quad i = 1, 2, \cdots, N-1.$$

where

$$\rho_k = \frac{q_1 \cdots q_k}{p_1 \cdots p_k}, \quad k = 1, 2, \cdots, N-1.$$

Problem 2. Consider the random walk Markov chain with transition probability matrix

$$P = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.3 & 0 & 0.7 & 0 \\ 0 & 0.3 & 0 & 0.7 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

The transition probability matrix corresponding to the nonabsorbing states is

$$Q = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} 0 & 0.7 \\ 0.3 & 0 \end{pmatrix} \end{array}.$$

Calculate the matrix inverse to $I - Q$, and from this determine

(a) the probability of absorption into state 0 from state 1;

(b) the mean time spent in each of states 1 and 2 prior to absorption.