Stat 461 HW # 7

Note Title

Problem 1. Consider the random walk Markov chain with transition probability matrix

$$
P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}
\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.3 & 0 & 0.7 & 0 \\ 0 & 0.3 & 0 & 0.7 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}
$$

The transition probability matrix corresponding to the nonabsorbing states is

$$
Q = \begin{array}{c} \\ 1 \\ 2 \end{array}
\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} 0 & 0.7 \\ 0.3 & 0 \end{pmatrix} \end{array}.
$$

Calculate the matrix inverse to $I - Q$, and from this determine

(a) the probability of absorption into state 0 from state 1;

(b) the mean time spent in each of states 1 and 2 prior to absorption.

Problem 2. Let Z be a random variable with geometric distribution with parameter p. That is

$$\mathbb{P}\{Z=k\}=p(1-p)^k, \quad k=0,1,2,\dots$$

(1) Compute the generating function $\phi_Z(s)$ of Z.

(2) From $\phi_Z(s)$, show that $\mathbb{E}Z=\frac{1-p}{p}$ and

$$\operatorname{Var} Z=\frac{1-p}{p^2}$$

Problem 3. Let $\phi(s)$ be the generating function of an offspring random variable ξ. Let Z be a random variable whose distribution is that of ξ, but conditional on $\xi>0$. That is

$$\mathbb{P}\{Z=k\}=\mathbb{P}\{\xi=k \mid \xi>0\} \quad k=1,2,\dots$$

Express the generating function for Z in terms of ϕ.