

Problem 1

A Markov chain X_0, X_1, \dots has the transition probability matrix

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.4 & 0.1 & 0.5 \end{pmatrix} \end{matrix}$$

Determine the limiting distributions.

Problem 2

A Markov chain X_0, X_1, \dots has the transition probability matrix

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.2 & 0.2 & 0.6 \\ 0.3 & 0.3 & 0.4 \end{pmatrix} \end{matrix}$$

What fraction of time, in the long run, does the process spend in state 1?

Problem 3

Determine the following limits in terms of the transition probability matrix $P = (p_{ij})$ and limiting distributions $\pi = (\pi_j)$ of a finite state regular Markov chain $\{X_n\}$

$$(a) \lim_{n \rightarrow \infty} P\{X_{n+1} = j \mid X_0 = i\}$$

$$(b) \lim_{n \rightarrow \infty} P\{X_n = k, X_{n+1} = j \mid X_0 = i\}$$