Math 310, Fall 2015
Instructor: Chris Skalit
Quiz 1

Name: ______________________ UIN: ______________________

1. (3 points) Find the reduced row echelon form of the following matrix: $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \end{bmatrix}$

Solution: We employ the following row operations:

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix} \quad \text{add } (-2) \text{ times Row 1 to Row 2}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix} \quad \text{multiply Row 2 by } 1/2$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{multiply Row 3 by } 1/3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{add (-1) Row 3 to Row 1}$$

2. (3 points) Consider the system of equations

$$\begin{aligned} x + ky &= 1 \\ 2x - y &= 0 \end{aligned}$$

For what value(s) of k is the system consistent? For what value(s) of k is the system inconsistent?

Solution: By adding (-2) times the first equation to the second, we obtain

$$\begin{aligned} x + ky &= 1 \\ 0 - (2k+1)y &= -2 \end{aligned}$$

If $k = -\frac{1}{2}$, we obtain the contradictory relation $0 = -2$ and thus have no solution. Otherwise, $2k + 1 \neq 0$, and we can solve the system explicitly.

(Quiz Contines on Reverse Side)

3. (4 points) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$. Does the vector $\mathbf{w} = \begin{bmatrix} 5 \\ 8 \\ -1 \end{bmatrix}$ belong to span $\{\mathbf{v}_1, \mathbf{v}_2\}$? If so, find scalars c_i such that $\mathbf{w} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2$.

Solution: Saying that $\mathbf{w}$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$ is equivalent to saying that the system of equations

$$\begin{aligned} c_1 + c_2 &= 5 \\ c_1 + 2c_2 &= 8 \\ c_1 - c_2 &= -1 \end{aligned}$$

has a solution. We can express this system via an augmented matrix $A = \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 1 & 2 & 8 \\ 1 & -1 & -1 \end{array}\right]$.

We compute $\text{rref}(A) = \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$, so $c_1 = 2$ and $c_2 = 3$.