Math 310 (35180), Fall 2015
Instructor: Chris Skalit
Quiz 10

Name: _____ UIN: _____

1. Let $\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix}$.

   (a) (2 points) Compute $\mathbf{x} \cdot \mathbf{y}$.

   Solution:

   $$\mathbf{x} \cdot \mathbf{y} = (1)(0) + (-2)(4) + (3)(-1) = -11$$

   (b) (2 points) If $\mathbf{v} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$, are $\mathbf{v}$ and $\mathbf{x} - \mathbf{y}$ orthogonal?

   Solution:

   $$\mathbf{v} \cdot (\mathbf{x} - \mathbf{y}) = 6(1 - 0) + 1(-2 - 4) + 0(3 - (-1)) = 0.$$

   Hence, $\mathbf{v}$ and $\mathbf{x} - \mathbf{y}$ are orthogonal.

   (c) (2 points) Compute $||\mathbf{x} + 2\mathbf{y}||$.

   Solution: $\mathbf{x} + 2\mathbf{y} = \begin{bmatrix} 1 \\ 6 \\ 1 \end{bmatrix}$, $||\mathbf{x} + 2\mathbf{y}|| = \sqrt{1^2 + 6^2 + 1^2} = \sqrt{38}$.

   (d) (2 points) Find the distance from $\mathbf{x}$ to $\mathbf{y}$.

   Solution: The distance from $\mathbf{x}$ to $\mathbf{y}$ is, by definition,
   $||\mathbf{x} - \mathbf{y}|| = \sqrt{(1 - 0)^2 + (-2 - 4)^2 + (3 - (-1))^2} = \sqrt{53}$.

   (e) (2 points) Find a **unit** vector $\mathbf{u}$ that points in the same direction as $\mathbf{x} + \mathbf{y}$.

   Solution: We renormalize $\mathbf{x} + \mathbf{y}$ by its length, so

   $$\mathbf{u} = \frac{\mathbf{x} + \mathbf{y}}{||\mathbf{x} + \mathbf{y}||} = \frac{1}{\sqrt{1 + 2^2 + 2^2}} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}.$$