Math 310 (35180), Fall 2015
Instructor: Chris Skalit
Quiz 11

Name: ______________________ UIN: ______________________

1. Let U be the subspace of $\mathbb{R}^3$ spanned by the vectors $\mathbf{u}_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Note that these vectors form an orthogonal basis for U.

   (a) (5 points) If $\mathbf{y} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$, compute the orthogonal projection $\text{proj}_U(\mathbf{y})$ of $\mathbf{y}$ onto U.

   Solution:

$$\begin{aligned} \text{proj}_U(\mathbf{y}) &= \frac{\mathbf{y}\cdot\mathbf{u}_1}{\mathbf{u}_1\cdot\mathbf{u}_1}\mathbf{u}_1 + \frac{\mathbf{y}\cdot\mathbf{u}_2}{\mathbf{u}_2\cdot\mathbf{u}_2}\mathbf{u}_2 \\ &= \left(\frac{3}{3}\right)\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \left(\frac{2}{2}\right)\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \end{aligned}$$

   (b) (1 point) Using your answer from part (a), compute the distance from $\mathbf{y}$ to U.
   Solution: This distance is merely

$$||\mathbf{y} - \text{proj}_U(\mathbf{y})|| = \sqrt{(-3-(-1))^2 + (1-0)^2 + (1-2)^2} = \sqrt{6}.$$

2. (4 points) Let W be the subspace of $\mathbb{R}^3$ spanned by $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{w}_2 = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$. Use the Gram-Schmidt process to find an orthogonal basis for W.

   Solution: We want to construct an orthogonal basis $\{\mathbf{u}_1, \mathbf{u}_2\}$. According to Gram-Schmidt, we can take

$$\mathbf{u}_1 = \mathbf{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } U_1 = \text{span}\{\mathbf{u}_1\}.$$

$$\mathbf{u}_2 = \mathbf{w}_2 - \text{proj}_{U_1}(\mathbf{w}_2) = \mathbf{w}_2 - \frac{\mathbf{w}_2\cdot\mathbf{u}_1}{\mathbf{u}_1\cdot\mathbf{u}_1}\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} - \frac{6}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$$