Math 310 (35180), Fall 2015
Instructor: Chris Skalit
Quiz 12

Name: ____________________ UIN: ____________________

1. (5 points) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$. Find all least squares solutions to the (inconsistent) system $A\mathbf{x} = \mathbf{b}$.

   Solution: A least squares solution $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ satisfies the equation $A^T A\mathbf{y} = A^T\mathbf{b}$, so we must solve

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

   We therefore see that $y_1 = 1$ and $y_2 = 0$.

2. (5 points) Consider the points $(0, 0)$, $(1, 1)$, and $(2, 1)$ in $\mathbb{R}^2$. Find the equation $y = a_0 + a_1 x$ of the best-fit line (in a least-squares sense) through these points.

   Solution: By plugging in the coordinates into the equation $y = a_0 + a_1 x$, we get

$$\begin{aligned} 0 &= a_0 + a_1(0) \\ 1 &= a_0 + a_1(1) \\ 1 &= a_0 + a_1(2) \end{aligned}$$

   Thus, to determine the coefficients a_0 and a_1, we seek the least squares solutions to $A\mathbf{a} = \mathbf{y}$ where $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$, $\mathbf{a} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. We solve

$$A^T A\mathbf{a} = A^T\mathbf{y} \iff \begin{bmatrix} 3 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

   and get $a_0 = 1/6$ and $a_1 = 1/2$.