Math 310, Fall 2015
Instructor: Chris Skalit
Quiz 3

Name: ______________________ UIN: ______________________

1. (6 points) Are the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 3 \\ -4 \\ 1 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -3 \\ 3 \\ 1 \end{bmatrix}$$

linearly independent? Why or why not? Show all work.

Solution: Let $A = \begin{bmatrix} 2 & -2 & 2 \\ 0 & 3 & -3 \\ 4 & -4 & 3 \\ 1 & 1 & 1 \end{bmatrix}$. After row reducing, we find that $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Since every column of $\text{rref}(A)$ is a pivot column, we conclude that the vectors are linearly independent.

2. (2 points) Without doing any computation, explain why the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \quad \mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

cannot be linearly independent.

Solution: If we let A be the matrix whose columns are the vectors $\mathbf{v}_i$, then we know that these vectors are linearly independent if and only if each of the four columns of $\text{rref}(A)$ contains a pivot. On the other hand, A has only three rows and so $\text{rref}(A)$ has at most three pivot points.

3. (2 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map. Suppose that $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^2$ such that

$$T(\mathbf{u}) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad T(\mathbf{v}) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Compute $T(2\mathbf{u})$ and $T(\mathbf{u} - \mathbf{v})$.

Solution: Using the linearity of T, we compute:

$$T(2\mathbf{u}) = 2T(\mathbf{u}) = 2\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$T(\mathbf{u} - \mathbf{v}) = T(\mathbf{u}) - T(\mathbf{v}) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$