**Math 310 (35180), Fall 2015**
**Instructor: Chris Skalit**
**Quiz 5**

Name: ______________________ UIN: ______________________

1. (4 points) If $A = \begin{bmatrix} 1 & 4 & 2 \\ 1 & 0 & 2 \\ 3 & 3 & 1 \end{bmatrix}$, compute $\det A$. Is $A$ invertible? Why or why not?

**Solution:** By using cofactor expansion along the second row, we get

$$\det A = (-1)\det\left(\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}\right) - 2\det\left(\begin{bmatrix} 1 & 4 \\ 3 & 3 \end{bmatrix}\right) = 20.$$

Note that $A$ is invertible since $\det A \neq 0$.

2. (4 points) If $A = \begin{bmatrix} 3 & 3 & 3 & 3 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 3 & 3 & 3 & 4 \end{bmatrix}$, compute $\det A$. **Hint:** First apply row reduction to $A$ before attempting cofactor expansion.

**Solution:** Dividing the first row by 3 gives $B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 3 & 3 & 3 & 4 \end{bmatrix}$.
Note that $3\det B = \det A$. Adding suitable multiples of the first row of $B$ to the rows below it gives $C = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Note that $\det C = \det B$. By using cofactor expansion along the first column, we get

$$\det C = 1 \cdot \det\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 1$$

Hence $\det A = 3\det B = 3\det C = 3$.

3. (2 points) Solve the following system of equations:

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & - & x_3 & = & 1 \\ 5x_1 & + & 4x_2 & + & 6x_3 & = & 1 \\ x_1 & + & x_2 & + & 3x_3 & = & 5 \end{array}$$

Use the following pieces of information and Cramer's Rule:

$$\det\left(\begin{bmatrix} 1 & 2 & -1 \\ 5 & 4 & 6 \\ 1 & 1 & 3 \end{bmatrix}\right) = -13 \qquad \det\left(\begin{bmatrix} 1 & 2 & -1 \\ 1 & 4 & 6 \\ 5 & 1 & 3 \end{bmatrix}\right) = 79$$

$$\det\left(\begin{bmatrix} 1 & 1 & -1 \\ 5 & 1 & 6 \\ 1 & 5 & 3 \end{bmatrix}\right) = -60 \qquad \det\left(\begin{bmatrix} 1 & 2 & 1 \\ 5 & 4 & 1 \\ 1 & 1 & 5 \end{bmatrix}\right) = -28$$

**Solution:** Our coefficient matrix is $A = \begin{bmatrix} 1 & 2 & -1 \\ 5 & 4 & 6 \\ 1 & 1 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$. If we denote by $A_i(\mathbf{b})$ the matrix obtained by replacing the $i$-th column of $A$ by $\mathbf{b}$, Cramer's rule says that $x_i = \frac{\det A_i(\mathbf{b})}{\det A}$. Hence we have

$$x_1 = -79/13 \qquad x_2 = 60/13 \qquad x_3 = 28/13.$$