Math 310 (35180), Fall 2015
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Quiz 6

Name: ______________________ UIN: ______________________

1. (7 points) Let $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 2 & 4 & 2 & 2 \\ 1 & 2 & -1 & 3 \end{bmatrix}$. Compute rref$(A)$ and use this information to write down bases for the column space Col(A) and the null space Nul(A).

   Solution: $\text{rref}(A) = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Since pivots occur in the first and third columns, we know that our column space has basis $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} \right\}$.

   The nullspace is just the solution set $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ to $A\mathbf{x} = \mathbf{0}$. From rref(A), we obtain relations $x_1 = -2x_2 - 2x_4$ and $x_3 = x_4$ where x_2 and x_4 are free. Hence

$$\text{Nul}(A) = \left\{ \begin{bmatrix} -2x_2 - 2x_4 \\ x_2 \\ x_4 \\ x_4 \end{bmatrix} : x_2, x_4 \in \mathbb{R} \right\} = \left\{ x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix} : x_2, x_4 \in \mathbb{R} \right\}$$

   Thus, Nul(A) has basis $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

2. (3 points) Let S be the collection of vectors in $\mathbb{R}^2$ defined by $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x = \pm y \right\}$. Show that S is **not** a subspace of $\mathbb{R}^2$ by explicitly finding two vectors $\mathbf{v}_1, \mathbf{v}_2 \in S$ whose sum lies outside of S.

   Solution: Note that $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ belong to S. However $\mathbf{v}_1 + \mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, which does not lie in S.