Math 310, Spring 2016
Instructor: Chris Skalit
Quiz 1

Name: ______________________ UIN: ______________________

1. (a) (5 points) Let $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 3 & 0 & 5 \\ 0 & 1 & 1 & 3 \end{bmatrix}$. Find the reduced-row-echelon form of A.

Solution: We show the reduction step-by-step:

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 3 & 0 & 5 \\ 0 & 1 & 1 & 3 \end{bmatrix} \quad \text{add } (-2) \text{ times R1 to R2}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 2 & 3 \\ 0 & 1 & 1 & 3 \end{bmatrix} \quad \text{multiply R2 by } (-1)$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -2 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix} \quad \text{add } (-2) \text{ times R2 to R1; add } (-1) \text{ times R2 and R3}$$

$$\begin{bmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 3 & 6 \end{bmatrix} \quad \text{multiply R3 by } 1/3$$

$$\begin{bmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \text{add 2 times R3 to R2; add } (-3) \text{ times R3 to R1}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

(b) (1 point) Use your answer to part (a) to find all solutions to the following system of equations:

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & - & x_3 & = & 1 \\ 2x_1 & + & 3x_2 & & & = & 5 \\ & & x_2 & + & x_3 & = & 3 \end{array}$$

Solution: The matrix in part (a) is the augmented matrix associated to this system. From its reduced row-echelon form, we get $x_1 = 1$, $x_2 = 1$, $x_3 = 2$

2. Consider the vectors $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ in $\mathbb{R}^3$. Compute:

   (a) (1 point) $\mathbf{x} + \mathbf{y}$

   Solution:

$$\mathbf{x} + \mathbf{y} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

   (b) (1 point) $2\mathbf{x} - 3\mathbf{y}$

   Solution:

$$2\mathbf{x} - 3\mathbf{y} = 2\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} - 3\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -11 \end{bmatrix}$$

3. (2 points) Determine all values of $\beta \in \mathbb{R}$ for which the system

$$\begin{array}{ccccc} x_1 & + & x_2 & = & 1 \\ x_1 & + & \beta x_2 & = & 0 \end{array}$$

has **NO** solutions.

Solution: The augmented matrix for this system reads $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \beta & 0 \end{bmatrix}$. Adding (-1) times the first row to the second gives $\begin{bmatrix} 1 & 1 & 1 \\ 0 & \beta - 1 & -1 \end{bmatrix}$. In particular, we have the relation, $(\beta - 1)x_2 = -1$. For the system to be solvable, we need the coefficient of x_2 to be nonzero. Otherwise, if $\beta - 1 = 0$, the system has no solutions.