Math 310 (33886), Spring 2016
Instructor: Chris Skalit
Quiz 10

Name: _____ UIN: _____

Let $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}$.

1. (1 point) Compute $\mathbf{x} \cdot \mathbf{y}$.

   Solution:
   $$\mathbf{x} \cdot \mathbf{y} = 1(0) + 2(3) - 3(-1) = 9$$

2. (1 point) Based on your answer to (1), are $\mathbf{x}$ and $\mathbf{y}$ orthogonal? Explain.

   Solution: No. Their dot product is nonzero.

3. (1 point) If $\mathbf{z} \in \mathbb{R}^3$ is such that $\mathbf{z} \cdot \mathbf{x} = 4$ and $\mathbf{z} \cdot \mathbf{y} = 3$, what is $\mathbf{z} \cdot (5\mathbf{x} + 2\mathbf{y})$?

   Solution: By the linearity of the dot product we have
   $$\mathbf{z} \cdot (5\mathbf{x} + 2\mathbf{y}) = 5(\mathbf{z} \cdot \mathbf{x}) + 2(\mathbf{z} \cdot \mathbf{y}) = 5(4) + 2(3) = 23$$

4. (2 points) What is the distance between $\mathbf{x}$ and $\mathbf{y}$?

   Solution: The distance between vectors, is by definition, given by
   $$||\mathbf{x} - \mathbf{y}|| = \sqrt{(1-0)^2 + (2-3)^2 + (-3-(-1))^2} = \sqrt{6}$$

5. (2 points) Find a unit vector which points in the same direction as $\mathbf{x} - 2\mathbf{y}$.

Solution: Let $\mathbf{v} = \mathbf{x} - 2\mathbf{y} = \begin{bmatrix} 1 \\ -4 \\ -1 \end{bmatrix}$. To get a unit vector in this direction, we rescale by $1/||\mathbf{v}||$:

$$\frac{1}{||\mathbf{v}||} \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{18}} \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$$

6. (3 points) Let $\mathbf{u} = \begin{bmatrix} a \\ b \\ 3 \end{bmatrix}$. If $\mathbf{u}$ is orthogonal to both $\mathbf{x}$ and $\mathbf{y}$, what are a and b?

Solution: From the relations $\mathbf{u} \cdot \mathbf{x} = \mathbf{u} \cdot \mathbf{y} = 0$, we obtain a system of equations

$$\begin{aligned} a + 2b - 9 &= \mathbf{u} \cdot \mathbf{x} &= 0 \\ 3b - 3 &= \mathbf{u} \cdot \mathbf{y} &= 0 \end{aligned}$$

And it's thus clear that $b = 1$ and $a = 7$.