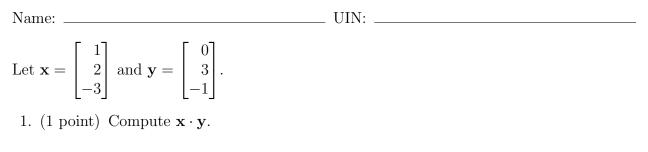
## Math 310 (33886), Spring 2016 Instructor: Chris Skalit Quiz 10



Solution:

$$\mathbf{x} \cdot \mathbf{y} = 1(0) + 2(3) - 3(-1) = 9$$

- (1 point) Based on your answer to (1), are x and y orthogonal? Explain.
  Solution: No. Their dot product is nonzero.
- 3. (1 point) If  $\mathbf{z} \in \mathbb{R}^3$  is such that  $\mathbf{z} \cdot \mathbf{x} = 4$  and  $\mathbf{z} \cdot \mathbf{y} = 3$ , what is  $\mathbf{z} \cdot (5\mathbf{x} + 2\mathbf{y})$ ?

Solution: By the linearity of the dot product we have

$$\mathbf{z} \cdot (5\mathbf{x} + 2\mathbf{y}) = 5(\mathbf{z} \cdot \mathbf{x}) + 2(\mathbf{z} \cdot \mathbf{y}) = 5(4) + 2(3) = 23$$

4. (2 points) What is the distance between  $\mathbf{x}$  and  $\mathbf{y}$ ?

Solution: The distance between vectors, is by definition, given by

$$||\mathbf{x} - \mathbf{y}|| = \sqrt{(1-0)^2 + (2-3)^2 + (-3-(-1))^2} = \sqrt{6}$$

5. (2 points) Find a unit vector which points in the same direction as  $\mathbf{x} - 2\mathbf{y}$ .

Solution: Let  $\mathbf{v} = \mathbf{x} - 2\mathbf{y} = \begin{bmatrix} 1\\ -4\\ -1 \end{bmatrix}$ . To get a unit vector in this direction, we rescale by  $1/||\mathbf{v}||$ :  $\frac{1}{||\mathbf{v}||} \begin{bmatrix} 1\\ -4\\ 1 \end{bmatrix} = \frac{1}{\sqrt{18}} \begin{bmatrix} 1\\ -4\\ 1 \end{bmatrix}$ 

6. (3 points) Let 
$$\mathbf{u} = \begin{bmatrix} a \\ b \\ 3 \end{bmatrix}$$
. If  $\mathbf{u}$  is orthogonal to both  $\mathbf{x}$  and  $\mathbf{y}$ , what are  $a$  and  $b$ ?

**Solution:** From the relations  $\mathbf{u} \cdot \mathbf{x} = \mathbf{u} \cdot \mathbf{y} = 0$ , we obtain a system of equations

$$a + 2b - 9 = \mathbf{u} \cdot \mathbf{x} = 0$$
  
$$3b - 3 = \mathbf{u} \cdot \mathbf{y} = 0$$

And it's thus clear that b = 1 and a = 7.