Math 310 (33886), Fall 2016
Instructor: Chris Skalit
Quiz 3

Name: ______________________ UIN: ______________________

1. (a) (4 points) Let $A = \begin{bmatrix} 2 & 0 & 4 \\ -2 & 1 & -3 \\ 1 & 1 & 3 \end{bmatrix}$ Find rref(A).

   Solution:

$$\begin{bmatrix} 2 & 0 & 4 \\ -2 & 1 & -3 \\ 1 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & -3 \\ 1 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

   (b) (1 point) Based on your answer to part (a), are the vectors $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 4 \\ -3 \\ 3 \end{bmatrix}$ linearly independent? Explain in one sentence.

   Solution: These vectors, which are the columns of A, are not linearly independent because rref(A) does not have a pivot in each column.

2. (2 points) Suppose that $S : \mathbb{R}^2 \to \mathbb{R}^3$ is linear and that $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - y \\ x + y \\ 4x \end{bmatrix}$. Write down the matrix B such that $S(\mathbf{v}) = B\mathbf{v}$ for all vectors $\mathbf{v} \in \mathbb{R}^2$.

   Solution: We have $S\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$ and $S\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, so $B = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ 4 & 0 \end{bmatrix}$

3. Consider the vectors $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{w}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in $\mathbb{R}^2$. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}$ is a linear transformation such that $T(\mathbf{w}_1) = 2$ and $T(\mathbf{w}_2) = 3$.

   (a) (2 points) Write $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ as a linear combination of $\mathbf{w}_1$ and $\mathbf{w}_2$.

   Solution: We solve the system $\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and find that $\alpha = 1, \beta = 3$.

   (b) (1 point) Using your answer to part (a), compute $T\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right)$.

Solution: From part (a), we have that $\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \mathbf{w}_1 + 3\mathbf{w}_2$, and so

$$T\left(\begin{bmatrix} 4 \\ 3 \end{bmatrix}\right) = T(\mathbf{w}_1 + 3\mathbf{w}_2) = T(\mathbf{w}_1) + 3T(\mathbf{w}_2) = 11.$$