Math 310 (33886), Fall 2016
Instructor: Chris Skalit
Quiz 4

Name: ______________________ UIN: ______________________

1. Compute the product (or say that the product doesn't exist):

   (a) (1 point) $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

   Solution: Undefined. The left matrix has 3 columns while the one of the right has only 2 rows.

   (b) (1 point) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

   Solution:

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

2. Let $A = \begin{bmatrix} -1 & -1 & 1 \\ 2 & 1 & 0 \\ -2 & -1 & 1 \end{bmatrix}$

   (a) (5 points) Compute A^{-1}.

   Solution: To find the inverse, we construct the 3×6 matrix $B = \begin{bmatrix} -1 & -1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ -2 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}$, which is comprised of the matrix A on the left and the 3×3 identity on the right. We compute

$$\operatorname{rref}\left(\begin{bmatrix} -1 & -1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ -2 & -1 & 1 & 0 & 0 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -2 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

   By reading off the right-hand side of this matrix, we see that $A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$.

(b) (1 point) Using your answer to (a), find the solution to $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

Solution: Since A is invertible, we can solve $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$:

$$A\mathbf{x} = \mathbf{b} \Rightarrow A^{-1}(A\mathbf{x}) = A^{-1}\mathbf{b} \Rightarrow \mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

3. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation defined by $T(\mathbf{x}) = B\mathbf{x}$ for $B = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(a) (1 point) Is T surjective (onto)? Explain in one sentence.

Solution: Yes. Each row of rref B has a pivot.

(b) (1 point) is T injective (one-to-one)? Explain in one sentence.

Solution: No. Not every column of rref B has a pivot.