Math 310 (33886), Spring 2016
Instructor: Chris Skalit
Quiz 7

Name: ________________________________ UIN: ________________________________

1. State the dimensions of the following spaces:

   (a) (1 point) The space $\mathbb{P}_2$ of all real polynomial functions having degree at most two.

   Solution: This space has a basis consisting of the polynomials $\{1, t, t^2\}$, so $\dim \mathbb{P}_2 = 3$.

   (b) (1 point) The column space of a 3×4 matrix A with $\dim(\operatorname{Nul} A) = 2$.

   Solution: This matrix has 4 columns, so by the rank-nullity theorem, $\dim(\operatorname{Nul} A) + \dim(\operatorname{Col} A) = 4$; hence $\dim(\operatorname{Nul} A) = 2$.

   (c) (1 point) The row space of a 9×9 matrix B such that $\dim(\operatorname{Col} B) = 7$.

   Solution: The row space and column space of a matrix have the same dimension, so $\dim(\operatorname{row} A) = 7$.

   (d) (1 point) The nullspace of a 4×3 matrix C whose columns are linearly independent.

   Solution: Recall that a matrix has linearly independent columns if and only if its nullspace consists only of $\mathbf{0}$. Hence, $\dim(\operatorname{Nul} C) = 0$.

2. Let V be the subspace of $\mathbb{R}^4$ having basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ where

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{b}_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

   (a) (2 points) If $\mathbf{v} \in V$ has coordinates $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$, what is $\mathbf{v}$?

   Solution: The above coordinate representation means precisely that

$$\mathbf{v} = (-1)\mathbf{b}_1 + 0 \cdot \mathbf{b}_2 + 2\mathbf{b}_3 = \begin{bmatrix} -3 \\ 2 \\ 1 \\ -1 \end{bmatrix}$$

(b) (4 points) Let $\mathbf{w} = \begin{bmatrix} -6 \\ 5 \\ 6 \\ 3 \end{bmatrix}$. Find $[\mathbf{w}]_{\mathcal{B}}$.

Solution: By definition $[\mathbf{w}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$ where the c_i satisfy the equation

$$\mathbf{w} = c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + c_3\mathbf{b}_3.$$

Thus, we must solve the equation

$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 5 \\ 6 \\ 3 \end{bmatrix}$$

whence we find that $c_1 = 1$, $c_2 = 2$, and $c_3 = 3$.