

Main Lemma: Let $R \rightarrow A \rightarrow \Lambda$ w/ A f.t. / R . Let $a \in R$ w/

- 1) $\text{ann}(a) = \text{ann}(a^2)$ in R and Λ
- 2) a in A is strictly standard.

Let $\bar{R} = R/a^2R$ etc. Suppose we have \bar{C} ft / \bar{R} w/ $\bar{R} \rightarrow \bar{A} \rightarrow \bar{C} \rightarrow \bar{\Lambda}$

Then $\exists B$ ft / R w/ $R \rightarrow A \rightarrow B \rightarrow \Lambda$ and $\pi^{-1}(h_{\bar{C}}) \subset h_B$
 [w/ $\pi: \Lambda \rightarrow \bar{\Lambda}$].

Note that from this lemma we have:

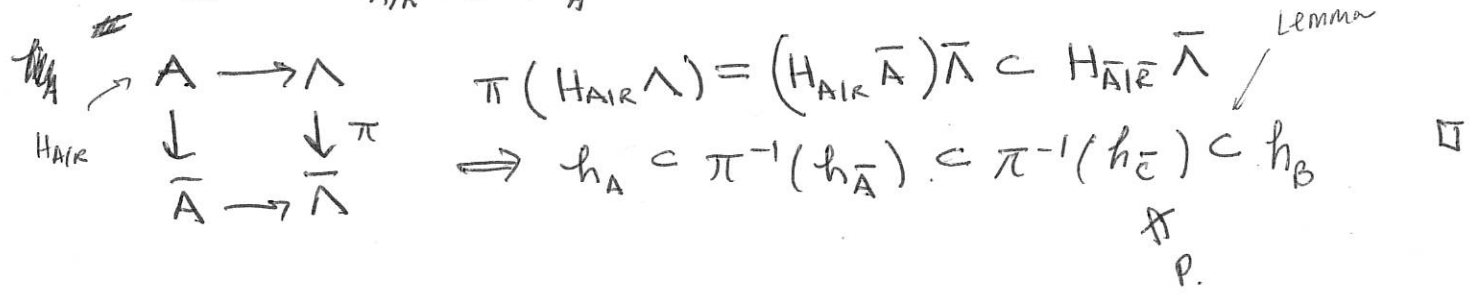
Resolvability of $\bar{R} \rightarrow \bar{A} \rightarrow \bar{\Lambda} \supset \bar{P}$ implies resolvability of $R \rightarrow A \rightarrow \Lambda \supset P$:

Indeed, let \bar{C} resolve $\bar{R} \rightarrow \bar{A} \rightarrow \bar{\Lambda} \supset \bar{P}$ (w/ $\bar{R} \rightarrow \bar{A} \rightarrow \bar{C} \rightarrow \bar{\Lambda}$ w/ $h_{\bar{B}} \subset h_{\bar{C}} \not\subset \bar{P}$ and hence $\pi^{-1}(h_{\bar{C}}) \not\subset P$.)

By the lemma, we have $R \rightarrow A \rightarrow B \rightarrow \Lambda$ w/ $\pi^{-1}(h_{\bar{C}}) \subset h_B$.

$H_{A/R} \bar{A} \subset H_{A/R} \bar{R}$ (stability of smoothness under base change).

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The Proof of Main Lemma 11.4 follows from two very technical calculations:

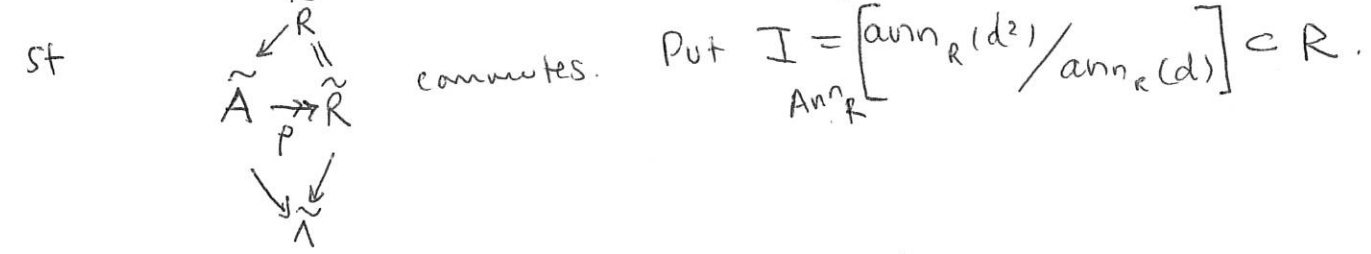
1) Lifting Lemma: Let $R \rightarrow \Lambda$ w/ $d \in R$ satisfying $\text{ann}(d) = \text{ann}(d^2)$ in R and Λ . Put $\bar{R} = R/d^2R$ $\tilde{R} = R/dR$. Suppose we have $\bar{R} \rightarrow \bar{C} \rightarrow \bar{\Lambda}$ \bar{C} ft / \bar{R} then $\exists D$ ft / R w/ $R \rightarrow D \rightarrow \Lambda$ and a diagram



w/ $\pi^{-1}(h_{\bar{C}}) \subset h_D$.

D) Desingularization Lemma (18.1)

Let $R \rightarrow \Lambda$ $d \in R$ satisfying $\text{ann}_\Lambda(d) = \text{ann}_\Lambda(d^2)$. Put $\tilde{R} = R/d^4R$, etc.
 Let $R \rightarrow A \rightarrow \Lambda$ (A ft R). $d \in A$ is strictly standard. Assume $p: \tilde{A} \rightarrow \tilde{R}$

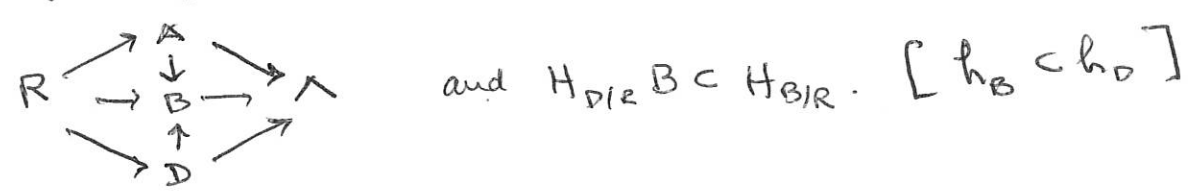


Then $\exists B$ st $R \rightarrow B \rightarrow \Lambda$ and $I B \subset H_{B/R}$.

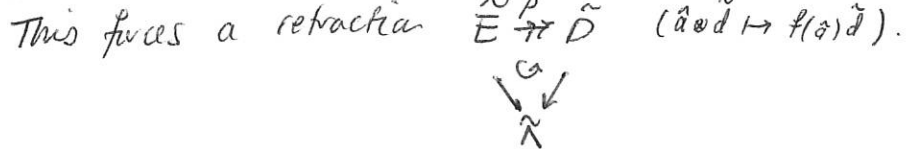
PROOF OF MAIN LEMMA.

CLAIM: (18.3) Let $R \rightarrow \Lambda$, $d \in R$ w/ $\text{ann}(d) = \text{ann}(d^2)$ in R AND Λ
 $R \rightarrow A \rightarrow \Lambda$ w/ $d \in A$ strictly standard. Put $\tilde{R} = R/d^4R$ etc.

Let $R \rightarrow D \rightarrow \Lambda$ and suppose there is a map factoring $\tilde{R} \rightarrow \hat{A} \xrightarrow{\hat{f}} \tilde{D} \rightarrow \tilde{\Lambda}$
 then there is a ft R alg B $R \rightarrow B \rightarrow \Lambda$ w/ maps



Pf: We construct B from $E = A \otimes_R D$. $d \in E$ is strictly standard.



We can apply the desingularization lemma to $D \rightarrow E \rightarrow \Lambda$ to get

B ft D w/ $D \rightarrow E \rightarrow B \rightarrow \Lambda$ and $\left[\frac{\text{ann}_D(d^2)}{\text{ann}_D(d)} \right] B \subset H_{B/D}$.

Fix $c \in H_{D/R}$ $R \rightarrow D_c$ is flat so $\text{ann}_R(t) = \text{ann}_{D_c}(t) \forall t \in R$ Call this ideal I

$$\text{ann}_R(d) = \text{ann}_R(d^2) \Rightarrow \text{ann}_{D_c}(d) = \text{ann}_{D_c}(d^2)$$

$$\Rightarrow I_c = D_c$$

$$\Rightarrow H_{B_c/D_c} = (1).$$

$$\Rightarrow B_c \text{ smooth / } D_c \text{ which is } R\text{-smooth}$$

$$\Rightarrow c \in H_{B/R} \Rightarrow H_{D/R}^B \subset H_{D/R} \quad \square$$

Pf of 11.3 | We have $R \rightarrow A \rightarrow \Lambda$

① $\text{ann}(a) = \text{ann}(a^2)$ in R and Λ

② a in A is standard.

$\bar{R} = R/a^2R$ etc. w/ a ft. \bar{R} alg \bar{C} satisfying $\bar{R} \rightarrow \bar{A} \rightarrow \bar{C} \rightarrow \bar{\Lambda}$

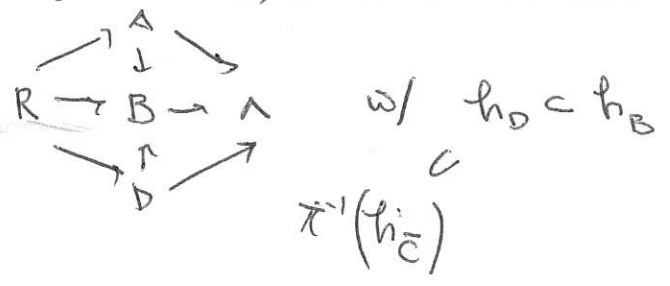
WTF: B st $R \rightarrow A \rightarrow B \rightarrow \Lambda$ w/ $\pi^{-1}(h_{\bar{C}}) \subset h_B$.

Put $d = a^4$. $\bar{R} = R/d^2R$ $\tilde{R} = R/dR$. we get $R \xrightarrow{H} D \rightarrow \Lambda$ [Lifting Lemma]

$$\begin{array}{ccc} \bar{R} & \rightarrow & \bar{C} \rightarrow \bar{\Lambda} \\ \downarrow f \downarrow \varphi & & \downarrow \\ \tilde{R} & \rightarrow & \tilde{D} \rightarrow \tilde{\Lambda} \end{array} \quad \begin{array}{l} = R/a^4R \\ \pi^{-1}(h_{\bar{C}}) \subset h_D \end{array}$$

Look at $\bar{R} \rightarrow \bar{A} \rightarrow \bar{C} \xrightarrow{\varphi} \tilde{D}$ this induces a map $\tilde{R} \rightarrow \tilde{A} \rightarrow \tilde{D} \rightarrow \tilde{\Lambda}$

~~CLAIM~~ Apply CLAIM 11.3 to $R \rightarrow D \rightarrow \Lambda$ and $R \rightarrow A \rightarrow \Lambda$ and get B ft R



[In the notation of 11.3 d is replaced w/ a]

□

Sketch of pf of desingularization

Fix a presentation $A = R[x_1, \dots, x_n]/I$

$P(x_i) = \tilde{y}_i \in \tilde{R}$ lift to $y_i \in R$. If $q(x) \in I \subset R[x]$ then $q(y_i) \in d^4 R$.

Say the image of d in A is represented by $P(x) = \Delta(f_1, \dots, f_r)[(f_i) \in I]$

in $R[x_1, \dots, x_n]$, $d - P(x) \in I \Rightarrow d - P(y) \in d^4 R$

$\tilde{u} - d - P(y) = d^4 t \Rightarrow P(y) = d(1 - d^3 t)$ $S \equiv 1 \pmod{d}$
 $S \in R$