

ARTIN APPROXIMATION THEOREM

RTG READING SEMINAR ON NERON POPESCU DESINGULARIZATION

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In [Art68, Theorem (1.2)], Artin proved (roughly speaking) that any system of convergent power series equations with coefficients on a valued field of characteristic zero has at least a convergent power series solution, provided it has at least a formal power series one. Later on (see [Art69, Theorem (1.10)]), he obtained an algebraic analogue of this result; namely, it was shown that any system of polynomial equations, with coefficients on an arbitrary field, and without independent term, has at least a polynomial solution, provided it has at least a formal power series one. Moreover, it was also asked by Artin (see [Art69, page 26] and [Art71]) whether the same result is true in case one considers a polynomial system of equations with coefficients on an arbitrary excellent henselian local ring, requiring as assumption the existence of a solution in its completion. Finally, in [Pop86] Popescu gave a positive answer to Artin's question, using as main tool what is nowadays called Neron-Popescu desingularization (see [Pop85]).

The main goal of this talk is to review with details Popescu's proof of Artin's approximation, following the exposition presented by Swan in [Swa98, Theorem 2.4]; if time permits, we also discuss applications and variants of this result.

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