1. (O’Neill 2.1.5) Prove that \( u \times v \neq 0 \) if and only if \( u \) and \( v \) are linearly independent, and show that \( ||u \times v|| \) is the area of a parallelogram with sides \( u \) and \( v \).

2. (O’Neill 2.1.8) Prove: The volume of the parallelepiped with sides \( u, v \) and \( w \) is \( \pm u \cdot v \times w \). (See the drawing and hint in the text.)

3. (O’Neill 2.2.2) Show that a curve has constant speed if and only if its acceleration is everywhere orthogonal to its velocity.
4. (O’Neill 2.2.3) Show that the curve \( \alpha(t) = (\cosh t, \sinh t, t) \) has arc length function \( s(t) = \sqrt{2} \sinh t \), and find a unit-speed reparametrization of \( \alpha \).

5. (O’Neill 2.2.8) Let \( Y \) be a vector field on a curve \( \alpha \). If \( \alpha(h) \) is a reparametrization of \( \alpha \), show that the reparametrization \( Y(h) \) is a vector field on \( \alpha(h) \), and prove the chain rule \( Y(h)' = h'Y'(h) \).

6. (O’Neill 2.3.2) Compute the Frenet apparatus \( \kappa, \tau, T, N, B \) of the unit-speed curve \( \beta(s) = (\frac{4}{5} \cos s, 1 - \sin s, -\frac{3}{5} \cos s) \). Show that this curve is a circle; find its center and radius.
7. (O’Neill 2.3.5) If $A$ is the vector field $\tau T + \kappa B$ on a unit-speed curve $\beta$, show that the Frenet formulas become

$$T' = A \times T,$$

$$N' = A \times N,$$

$$B' = A \times B.$$ 

8. (O’Neill 2.4.3) The curve $\alpha(t) = (t \cos t, t \sin t, t)$ lies on a double cone and passes through the vertex at $t = 0$.

(a) Find the Frenet apparatus of $\alpha$ at $t = 0$.

(b) Sketch the curve for $-2\pi \leq t \leq 2\pi$, showing $T$, $N$, $B$ at $t = 0$. 


9. (O’Neill 2.4.4) Show that the curvature of a regular curve in $\mathbb{R}^3$ is given by

$$\kappa^2 \nu^4 = ||\alpha''||^2 - \left(\frac{d\nu}{dt}\right)^2.$$ 

10. (O’Neill 2.4.9) If $\alpha$ is a curve with $\kappa > 0$ and $\tau$ both constant, show that $\alpha$ is a circular helix.