**Midterm Exam – Math 446 – Spring 2010**

Do at least 5 of the following problems. Specify which ones should be counted (at 20 points each) for the exam score.

**1.** Let $X$ be a topological space. Let $\Delta = \{(x, x) \mid x \in X\}$ be the "diagonal" in $X \times X$. Prove that $X$ is Hausdorff if and only if $\Delta$ is closed.

**2.** Suppose that $A$ is a connected subspace of a space $X$. Suppose that $A \subset B \subset \overline{A}$. Prove that $B$ is connected.

**3.** Prove that the continuous image of a compact space is compact.

**4.** Let $f : X \twoheadrightarrow Y$ be a surjective continuous map from a topological space $X$ to a topological space $Y$. Show that if $f$ is open then it is a quotient map. Give an example of a quotient map which is not open.

**5.** Let $A$ be a subspace of a metric space $X$. Prove that every limit point of $A$ is the limit of a convergent sequence of points of $A$.

**6.** Prove that there is no continuous surjection from $[0, 1]$ to $(0, 1)$.

**7.** Let $X$ be a connected space and $f : X \to X$ be a continuous map. Suppose that every point $x \in X$ has a neighborhood $U$ such that either:

- the restriction $f|_U$ is the identity map of $U$; or
- $f(U) \cap U = \emptyset$

Prove that if $f$ has a fixed point then $f$ must be the identity map of $X$.

**8.** Let $\mathbb{Z}$ act on $\mathbb{C}^* \doteq \mathbb{C} - \{0\}$ by $n \cdot z = 2^n z$. Prove that the quotient space $\mathbb{C}^*/\mathbb{Z}$ is homeomorphic to the torus $S^1 \times S^1$. Show that, with the same action, $\mathbb{C}/\mathbb{Z}$ is not Hausdorff.

**9.** Prove that the product of two connected spaces is connected.

**10.** Suppose that $A$ and $B$ are disjoint closed sets in a compact Hausdorff space $X$. Show there exist disjoint open sets $U$ and $V$ with $A \subset U$ and $B \subset V$.